Chapter 7

Matrices and Determinants

Definition:

A matrix is a rectangular array or arrangement of entries or elements displayed in rows and columns put within a square bracket [ ] .

Order or Size of the matrix

- If a matrix A has m rows and n columns then the order or size of the matrix A is defined to be $m \times n$ (read as m by n).

Types of Matrices

1. ROW MATRIX

A matrix having only one row is called a row matrix.

Example

- 1. $A = [a_{ij}]_{1\times n}$
- 2. $A = [1 \quad 2 \quad 3 \quad 4]_{1\times 4}$

2. COLUMN MATRIX

A matrix having only one column is called a column matrix .

Example

- 1. $A = [a_{ij}]_{m\times 1}$
- 2. $\mathrm{A} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1}$

3. ZERO MATRIX

A matrix $A = [a_{ij}]_{m\times n}$ is said to be a zero matrix or null matrix or void matrix denoted by 0 if $a_{ij} = 0$ for all values of $1 \leq i \leq m$ and $1 \leq j \leq n$.

Example

- 1. $A = [0]_{1\times 1}$
- 2. $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$

4. SQUARE MATRIX

A matrix in which number of rows is equal to the number of columns, is called a square matrix.

Example

- 1. $A = [a_{ij}]_{n \times n}$
- 2. $\mathrm{B} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & f \end{bmatrix}_{3 \times 3}$

5. DIAGONAL MATRIX

A square matrix $A = [a_{ij}]_{n\times n}$ is called a diagonal matrix if $a_{ij} = 0$ whenever $i \neq j$.

Example

- 1. $A = \begin{bmatrix} r & 0 \\ 0 & s \end{bmatrix}_{2\times 2}$

- 2. $B = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & 0.5 \end{bmatrix}_{3\times 3}$

6. SCALAR MATRIX

A diagonal matrix whose entries along the principal diagonal are equal is called a **scalar matrix.**

Example

- 1. $A = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}_{2\times 2}$

- 2. $B = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}_{3\times 3}$

7.UNIT MATRIX

A square matrix in which all the diagonal entries are 1 and the rest are all zero is called a unit matrix

Example

- 1. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

- 2. $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$

8. UPPER TRIANGULAR MATRIX

A square matrix is said to be an upper triangular matrix if all the elements below the main diagonal are zero.

Example

- 1. $A = \begin{bmatrix} 2 & 3 \\ 0 & 6 \end{bmatrix}_{2\times 2}$

- 2. $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 9 \end{bmatrix}_{3\times 3}$

9. LOWER TRIANGULAR MATRIX

A square matrix is said to be a lower triangular matrix if all the elements above the main diagonal are zero.

Example

- 1. $A = \begin{bmatrix} 2 & 0 \\ 3 & 6 \end{bmatrix}_{2\times 2}$

- 2. $B = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ -5 & 8 & 9 \end{bmatrix}_{3\times 3}$

10. TRIANGULAR MATRIX

A square matrix which is either upper triangular or lower triangular is called a **triangular matrix.**

Example

- 1 $A = \begin{bmatrix} 4 & 3 & 2 \\ 0 & 7 & 8 \\ 0 & 0 & 9 \end{bmatrix}_{3\times3}$ → **Upper triangular matrix**

- 2 $B = \begin{bmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 8 & -5 & 7 \end{bmatrix}_{3\times3}$ → **Lower triangular matrix**

Thank you