MEI Mathematics Education Innovation

Lesson 3: Staying in balance

Goals

- Interpret balance diagrams (orally and in writing) and write equations that represent relationships between the weights on a balanced balance diagram.

- Use balanced balances to explain (orally and in writing) how to find solutions to equations of the form $x + p = q$ or $px = q$.

Learning Targets

- I can compare doing the same thing to the weights on each side of a balanced balance to solving equations by subtracting the same amount from each side or dividing each side by the same number.

- I can explain what a balanced balance and a true equation have in common.

- I can write equations that could represent the weights on a balanced balance.

Lesson Narrative

The goal of this lesson is for students to understand that we can generally approach $p + x = q$ by subtracting the same thing from each side and that we can generally approach $px = q$ by dividing each side by the same thing. This is accomplished by considering what can be done to a balance to keep it balanced.

Students are solving equations in this lesson in a different way than they did in the previous lessons. They are reasoning about things one could "do" to balances while keeping them balanced alongside an equation that represents a balance, so they are thinking about "doing" things to each side of an equation, rather than simply thinking "what value would make this equation true."

Alignments

Addressing

- Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

- Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Building Towards

- Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Instructional Routines

- Collect and Display
- Compare and Connect
- Notice and Wonder
- Think Pair Share

Student Learning Goals

Let's use balanced balances to help us solve equations.

3.1 Hanging Around

Warm Up: 10 minutes

Students encounter and reason about a concrete situation, balances with equal and unequal weights on each side. They then see diagrams of balanced and unbalanced balances and think about what must be true and false about the situations. In subsequent activities, students will use the balance diagrams to develop general strategies for solving equations.

Instructional Routines

- Notice and Wonder

Launch

Display the photo of socks and ask students, “What do you notice? What do you wonder?”

Give students 1 minute to think about the picture. Record their responses for all to see.

Things students may notice:

- There are two pink socks and two blue socks.
- The socks are clipped to either ends of two clothes hangers. The hangers are hanging from a rod.
- The hanger holding the pink socks is level; the hanger holding the blue socks is not level.

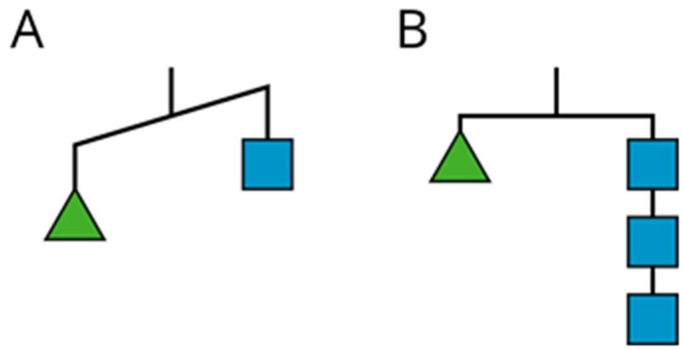
Things students may wonder:

- Why is the hanger (balance) holding the blue socks not level?
- Is something inside one of the blue socks to make it heavier than the other sock?
- What does this picture have to do with maths?

Use the word "balanced" to describe the hanger (balance) on the left and "unbalanced" to describe the hanger (balance) on the right. Tell students that the hanger (balance) on the left is balanced because the two pink socks have an equal weight, and the hanger (balance) on the right is unbalanced because one blue sock is heavier than the other. Tell students that they will look at a diagram that is like the photo of socks, except with more abstract shapes, and they will reason about the weights of the shapes.

Give students 3 minutes of quiet work time followed by whole-class discussion.

Student Task Statement



For diagram A, find:

1. One thing that *must* be true
2. One thing that *could* be true or false
3. One thing that *cannot possibly* be true

For diagram B, find:

1. One thing that *must* be true
2. One thing that *could* be true or false
3. One thing that *cannot possibly* be true

Student Response

Answers vary. Sample responses:

Diagram A:

1. The triangle is heavier than the square.
2. The triangle could weigh 10 grams and the square could weigh 6 grams.
3. The square and the triangle weigh the same.

Diagram B:

1. One triangle weighs the same as three squares.
2. The triangle weighs three kilograms and each square weighs one kilogram.
3. One square is heavier than the triangle.

Activity Synthesis

Ask students to share some things that must be true, could be true, and cannot possibly be true about the diagrams. Ask them to explain their reasoning. The purpose of this discussion is to understand how the balance diagrams work. When the diagram is balanced, there is equal weight on each side. For example, since diagram B is balanced, we know that one triangle weighs the same as three squares. When the diagram is unbalanced, one side is heavier than the other. For example, since diagram A is unbalanced, we know that one triangle is heavier than one square.

3.2 Match Equations and Balances

15 minutes

Students are presented with four balance diagrams and are asked to match an equation to each balance. They analyse relationships and find correspondences between the two representations. Then students use the diagrams and equations to find the unknown value in each diagram. This value is a solution of the equation.

Notice that the balances (and equations) for x and z are identical except that the variable appears on alternate sides of the equal sign. It may be obvious to some students that $3x = 6$ and $6 = 3z$ mean the same thing mathematically, but we know that in early KS3 many students do not have a well-developed understanding of what the equal sign means. So it is worth spending a little time to make explicit that these equations each have the solution 2. When we are writing an equation, it means the same thing if the two sides are swapped. Generally, $a = b$ means the same thing as $b = a$ where a and b represent any mathematical expression.

Instructional Routines

- Collect and Display
- Think Pair Share

Launch

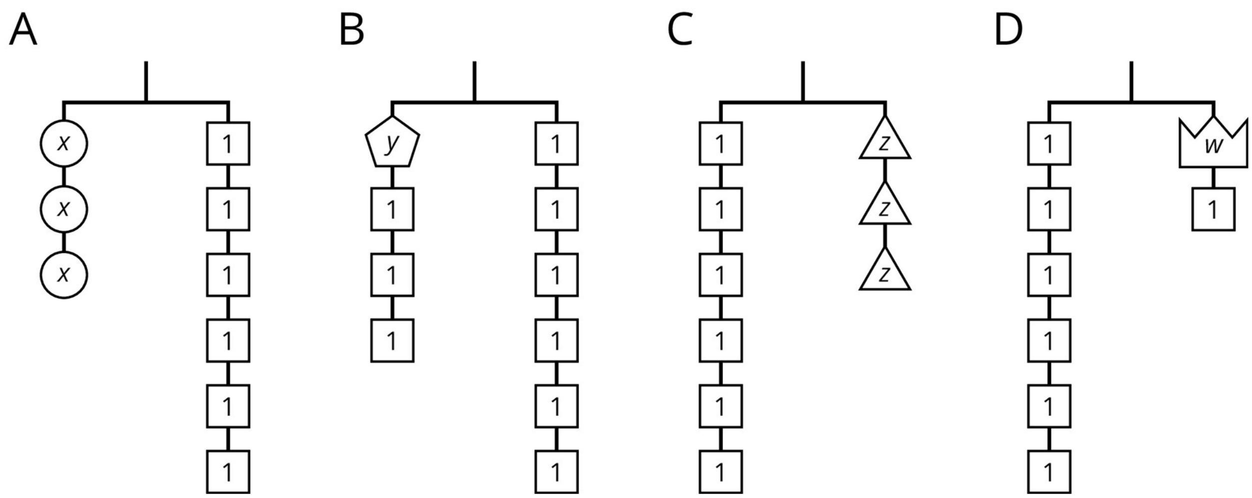
Display the diagrams and explain that each square labelled with a 1 weighs 1 unit, and each shape labelled with a letter has an unknown weight.

Arrange students in groups of 2. Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Conversing, Representing, Writing: Collect and Display. While pairs are working, circulate and listen to student talk about the balances and the equations. Write down common phrases and terms you hear students say about each representation (e.g., level, equal, the same as, balanced, tilted, more than, less than, unbalanced, grouping). Write the students' words with the matched representation on a visual display and refer to it during the synthesis. This will help students connect everyday words and mathematical language for

use during their paired and whole-group discussions.
Design Principle(s): Support sense-making

Student Task Statement



1. Match each balance to an equation. Complete the equation by writing x, y, z, or w in the empty box.

   $\square + 3 = 6$

   $3 \times \square = 6$

   $6 = \square + 1$

   $6 = 3 \times \square$

2. Find a solution to each equation. Use the balances to explain what each solution means.

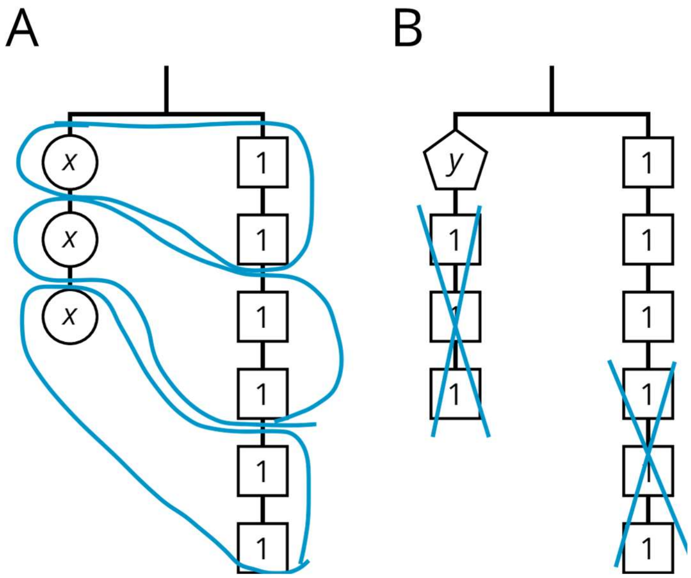
Student Response

1. A: $3 \times x = 6$, B: $y + 3 = 6$, C: $6 = 3 \times z$, D: $6 = w + 1$

2. A: x is 2, each circle weighs the same as 2 squares. B: y is 3, the pentagon weighs as much as 3 squares. C: z is 2, the z shape weighs the same as 2 squares. D: w is 5, the w shape weighs as much as 5 squares.

Activity Synthesis

Demonstrate two specific things for these specific examples: grouping the shapes on each side, and removing shapes from each side. In each case, the solution represents the weight of one shape. When you are done demonstrating, your diagrams might look like this:



Consider asking some of the following questions:

- “Explain how you know from looking at a balance that it can be represented by an equation involving addition.”
- “Explain how you know from looking at a balance that it can be represented by an equation involving multiplication.”
- “What are some moves that ensure that a balanced balance stays balanced?”

3.3 Connecting Diagrams to Equations and Solutions

15 minutes

This activity continues the work of using a balanced balance to develop strategies for solving equations. Students are presented with balanced balances and are asked to write equations that represent them. They are then asked to explain how to use the diagrams, and then the equations, to reason about a solution. Students notice the structure of equations and diagrams and find correspondences between them and between solution strategies.

Instructional Routines

- Compare and Connect
- Think Pair Share

Launch

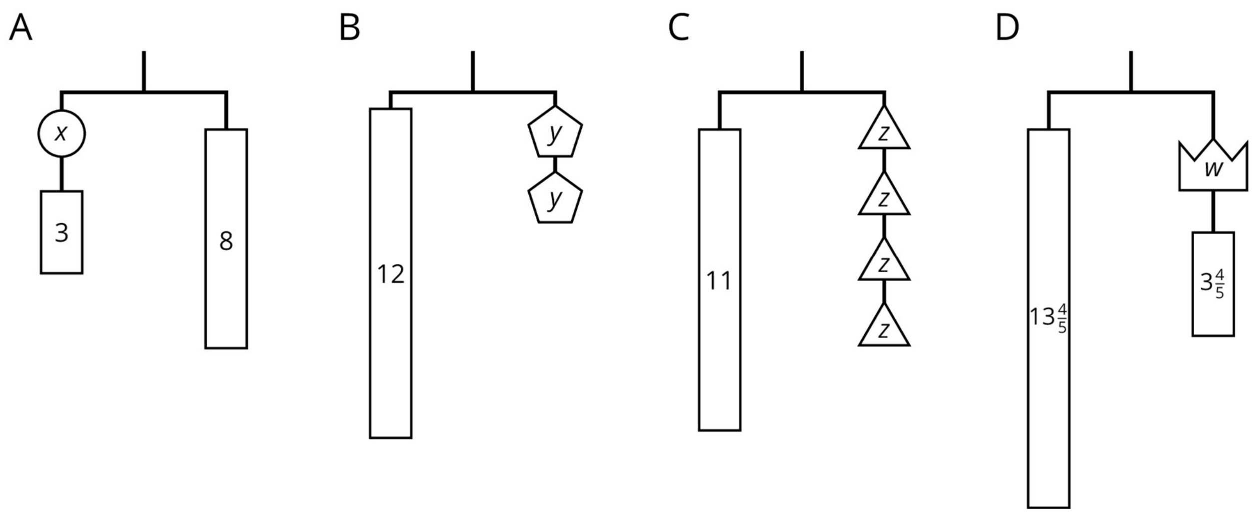
Draw students' attention to the diagrams in the task statement. Ensure they notice that the balances are balanced and that each object is labelled with its weight. Some weights are labelled with numbers but some are unknown, so they are labelled with a variable.

Keep students in the same groups. Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Representation: Internalise Comprehension. Demonstrate and encourage students to use colour coding and annotations to highlight connections between representations. See activity synthesis from the previous activity for an example.
Supports accessibility for: Visual-spatial processing

Student Task Statement

Here are some balanced balances. Each piece is labelled with its weight.



For each diagram:

1. Write an equation.
2. Explain how to reason with the *diagram* to find the weight of a piece with a letter.
3. Explain how to reason with the *equation* to find the weight of a piece with a letter.

Student Response

1. A: $x + 3 = 8$, B: $12 = 2y$, C: $11 = 4z$, D: $13\frac{4}{5} = w + 3\frac{4}{5}$

2.

   a. $x = 5$. Together x and 3 weigh 8, so x weighs 5.

   b. $y = 6$. 12 is twice the weight of y, so y weighs half of 12.

   c. $z = \frac{11}{4}$. 11 is 4 times the weight of z, so z weighs a quarter of 11.

   d. $w = 10$. Together w and $3\frac{4}{5}$ weight $13\frac{4}{5}$, so w weighs 10.

3.

a. Subtracting 3 from each side of the equation leaves $x = 5$.

b. The right side of the equation is equal to $2y$. After dividing each side of the equation by 2, the equation is $6 = y$.

c. Dividing each side of the equation by 4 leaves $\frac{11}{4} = z$.

d. Subtracting $3\frac{4}{5}$ from each side of the equation leaves $10 = w$.

Are You Ready for More?

When you have the time, visit the site https://solveme.edc.org/Mobiles.html to solve some trickier puzzles that use balance diagrams like the ones in this lesson. You can even build new ones. (If you want to do this during class, check with your teacher first!)

Activity Synthesis

Invite students to demonstrate, side by side, how they reasoned with both the diagram and the equation. For example, diagram A can be shown next to the equation $x + 3 = 8$. Cross out a piece representing 3 from each side, and write $x + 3 - 3 = 8 - 3$, followed by $x = 5$. Repeat for all four diagrams. For the diagrams represented by a multiplication equation, show dividing each side into equal-sized groups.

We want students to walk away with two things:

- An instant recognition of the structure of equations of the form $x + p = q$ and $px = q$ where p and q are specific, given numbers.
- A visual representation in their mind that can be used to support understanding of why for equations of the form $x + p = q$, you can subtract p from both sides, and for equations of the form $px = q$, you can divide both sides by p to find the solution.

Representing, Listening, Speaking: Compare and Connect. Use this routine to help students consider audience when preparing to share their work. Ask students to prepare a visual display that shows their reasoning about the diagram and equation. Some students may wish to add notes or details to their displays to help communicate their thinking. Invite students to share their displays with a partner, and discuss "What is the same and what is different?" Listen for the ways students describe the correspondences between the structures of the equations and diagrams, and between their solution strategies. This will help students use mathematical language to connect equations with diagrams during the synthesis.
Design Principle(s): Optimise output; Maximise meta-awareness

Lesson Synthesis

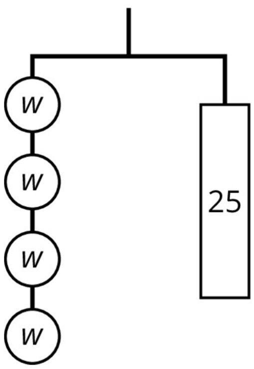
Display the two equations $5x = 8$ and $5 + x = 8$. Ask students to draw a balance to match each equation. Then have them work with a partner to solve the equation alongside finding the unknown value on the balance. Ask students to compare the two strategies and discuss how they are alike and how they are different.

3.4 Weight of the Circle

Cool Down: 5 minutes

Student Task Statement

Here is a balanced balance.



1. Write an equation representing this balance.

2. Find the weight of one circle. Show or explain how you found it.

Student Response

1. $4w = 25$

2. $\frac{25}{4}$ or $6\frac{1}{4}$

Student Lesson Summary

A balance stays balanced when the weights on both sides are equal. We can change the weights and the balance will stay balanced as long as both sides are changed in the same way. For example, adding 2 kg to each side of a balanced balance will keep it balanced. Removing half of the weight from each side will also keep it balanced.

An equation can be compared to a balanced balance. We can change the equation, but for a true equation to remain true, the same thing must be done to both sides of the equal sign. If we add or subtract the same number on each side, or multiply or divide each side by the same number, the new equation will still be true.

This way of thinking can help us find solutions to equations. Instead of checking different values, we can think about subtracting the same amount from each side or dividing each side by the same number.

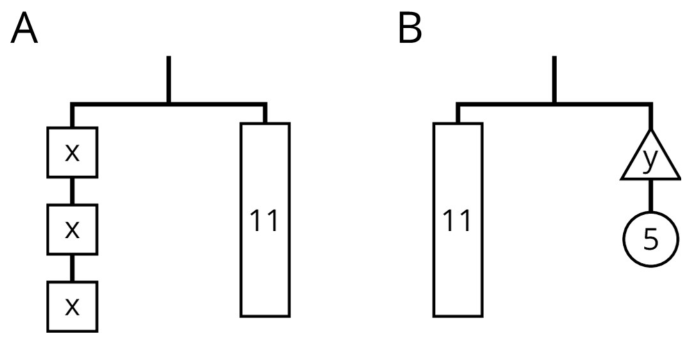


Diagram A can be represented by the equation $3x = 11$.

If we break the 11 into 3 equal parts, each part will have the same weight as a block with an x.

Splitting each side of the balance into 3 equal parts is the same as dividing each side of the equation by 3.

- $3x$ divided by 3 is x.
- 11 divided by 3 is $\frac{11}{3}$.
- If $3x = 11$ is true, then $x = \frac{11}{3}$ is true.
- The solution to $3x = 11$ is $\frac{11}{3}$.

Diagram B can be represented with the equation $11 = y + 5$.

If we remove a weight of 5 from each side of the balance, it will stay in balance.

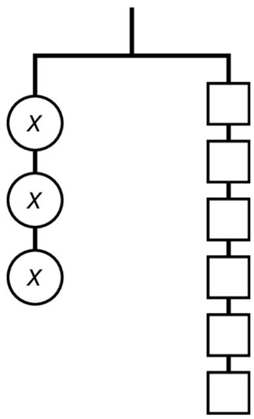
Removing 5 from each side of the balance is the same as subtracting 5 from each side of the equation.

- $11 - 5$ is 6.
- $y + 5 - 5$ is y.
- If $11 = y + 5$ is true, then $6 = y$ is true.
- The solution to $11 = y + 5$ is 6.

Lesson 3 Practice Problems

1. **Problem 1 Statement**

Select **all** the equations that represent the balance.



a. $x + x + x = 1 + 1 + 1 + 1 + 1 + 1$

b. $x \times x \times x = 6$

c. $3x = 6$

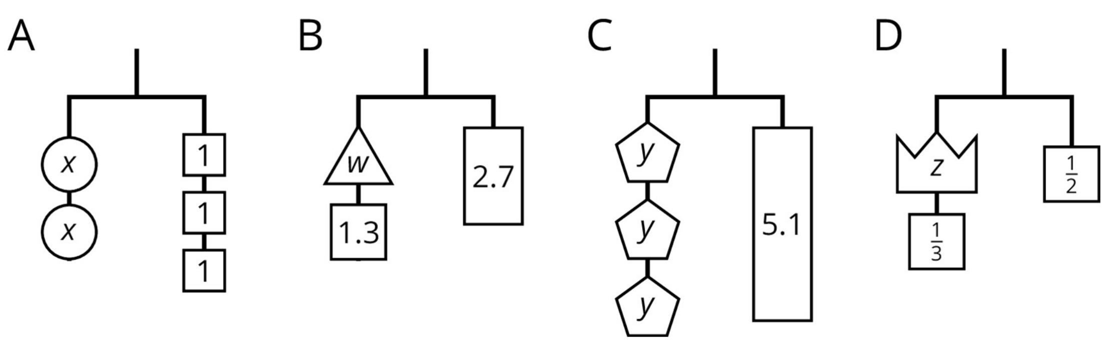
d. $x + 3 = 6$

e. $x \times x \times x = 1 \times 1 \times 1 \times 1 \times 1 \times 1$

Solution ["A", "C"]

2. **Problem 2 Statement**

Write an equation to represent each balance.



Solution

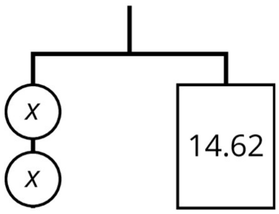
a. $2x = 3$ (or equivalent)

b. $w + 1.3 = 2.7$ (or equivalent)

c. $3y = 5.1$ (or equivalent)

d. $z + \frac{1}{3} = \frac{1}{2}$ (or equivalent)

3. **Problem 3 Statement**



a. Write an equation to represent the balance.

b. Explain how to reason with the balance to find the value of x.

c. Explain how to reason with the equation to find the value of x.

Solution

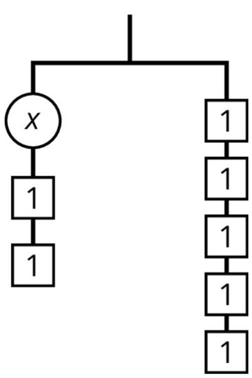
a. $2x = 14.62$

b. Each x can be grouped with half of the other side, so that means x is half of 14.62 or 7.31.

c. 14.62 is twice x, so x must be 7.31, since $2 \times (7.31) = 14.62$.

4. **Problem 4 Statement**

Andre says that x is 7 because he can move the two 1s with the x to the other side.



Do you agree with Andre? Explain your reasoning.

Solution

Andre is not correct. Each 1 on the left balances with a 1 on the right. So taking away the two 1s on the left only leaves the balance balanced if two 1s are removed on the right. This leaves x on the left and three 1s on the right, so $x = 3$.

5. **Problem 5 Statement**

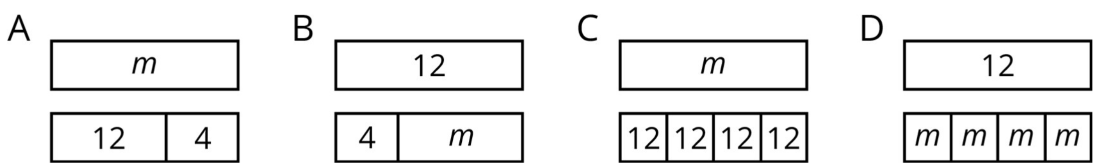
Match each equation to one of the diagrams.

a. $12 - m = 4$

b. $12 = 4 \times m$

c. $m - 4 = 12$

d. $\frac{m}{4} = 12$



Solution

- $12 - m = 4$ matches B
- $12 = 4 \times m$ matches D
- $m - 4 = 12$ matches A
- $\frac{m}{4} = 12$ matches C

(From Algebra 1, Lesson 1.1)

6. **Problem 6 Statement**

The area of a rectangle is 14 square units. It has side lengths x and y. Given each value for x, find y.

a. $x = 2\frac{1}{3}$

b. $x = 4\frac{1}{5}$

c. $x = \frac{7}{6}$

Solution

a. $y = 6$ $(14 \div 2\frac{1}{3} = 14 \div \frac{7}{3}$, and $14 \times \frac{3}{7} = 6)$

b. $y = 3\frac{1}{3}$ $(14 \div 4\frac{1}{5} = 14 \div \frac{21}{5}$, and $14 \times \frac{5}{21} = 3\frac{1}{3})$

c. $y = 12$ $(14 \div \frac{7}{6} = 14 \times \frac{6}{7} = 12)$

7. **Problem 7 Statement**

Lin needs to save up £20 for a new game. How much money does she have if she has saved each percentage of her goal. Explain your reasoning.

a. 25%

b. 75%

c. 125%

Solution

a. £5

b. £15

c. £25. Reasoning varies. Sample reasoning: