



**Grade / Age:** 10-14, 14-18 ages  
**Topic:** Mathematics, geometry, STEAM  
**Subject area:** plane isometries  
**Keywords:** friezes, rosettes  
**Single/ team work:** both  
**Language:** (English or Local) English  
**Duration:** 2 hours

**Description of the Task:**

More interesting than simple symmetries are groups of symmetries. Among these we will deal with the friezes and rosettes.

**1. Friezes**

Friezes are two-dimensional designs that repeat in one direction. The pattern you draw can be any pattern you like, you can repeat it as many times as you like. It can also contain symmetries. There are seven types of line patterns.

Download the Kali program to your computer.

<https://www.geometrygames.org/Archive/Kali/index.html>

The program can be used to draw frieze patterns. Let's look at all 7 patterns, how do they differ? What are they the same? What is the symmetry of the basic pattern in some cases?

Make frieze patterns by folding paper. For inspiration, watch this video:

<https://www.youtube.com/watch?v=LVxTESzflt8>

You can also create frieze patterns in GeoGebra using offset and mirror.

Collect buildings, handicrafts (embroideries) with frieze patterns.

**2. Rosettes**

Rosettes (rose windows) are named after the rose windows in churches.

Search the internet for buildings with rose windows. Many flowers and fruit cut in half have a similar symmetry. Collect plants with rosette symmetry from nature.

Even car hubcaps have rosette symmetry. Look for as many objects as possible in your environment that have rosette symmetry.

You can also use the Kali program to draw rosettes. Examine the resulting patterns, how do they differ? What are they the same? Which ones contain axial reflections and which ones do not?

Use folding to create images with rosette symmetry. For inspiration, see the following video. <https://www.youtube.com/watch?v=zM9O4alNYkc&t=37s>

You can also draw rosettes on paper with a ruler and a pencil, and colour them.

These patterns are also called mandalas.

You can also create rosettes in GeoGebra.

### Solutions of the Task:

The class of isometry groups containing a single translation are frieze groups. There are exactly seven ways of creating (infinite) linear patterns (friezes) which are generated by the (infinite) repetition of one motif.

Group code	Pattern
p111	...LLL...
p112	...NNN...
p1m1	...DDD...
p1a1	...bpbpbp..
pm11	...AAA...
pmm2	...HHH...
pma2	...ΛVΛ...

The explanation for codes is the following:

Letter  $p$  at position 1 comes from the word pattern.

At position 2 there is a letter  $m$  if the pattern contains reflection symmetry with an axis perpendicular to the direction of translation. Otherwise we write  $1$ .

At position 3 there are letters  $m$  or  $a$  if the pattern contains reflection symmetry with an axis parallel to the direction of translation, or a glide reflection, respectively.

At position 4 comes the order of rotation. It can be proven that frieze patterns can only have rotation centres of order two.

Friezes in GeoGebra: <https://www.geogebra.org/classic/hkuvwgxt>

<https://www.geogebra.org/classic/am4s96dx>

Frieze symmetry can be seen on many buildings and is very common in embroidery patterns. An interesting fact:

Decorated mud houses - Burkina Faso

The country of honourable people (the meaning of its name), Burkina Faso is located in West Africa. Near the border with Ghana is a 1.2-hectare circular village called Tiébélé. The village is inhabited by the Kassena ethnic group, and is famous for the richly decorated mud houses with drawings. Even in Africa, Burkina Faso is one of the poorest countries in the world, but it boasts a rich culture. Traditionally, from the 16th century onwards, the women of the village worked together to decorate the walls. Wood, soil, cow dung and straw are used to build houses. The houses are built without windows, with only sunlight filtering in through the door and possibly through a gap. Once the house is built, the women draw on the walls with coloured mud or chalk. The symbols are drawn from everyday life and religion. The walls are then sanded smooth with stone and a natural coat of varnish is applied to the entire wall.



Exact definition of rosette (rose window) group:

The Rosette (rose window) group is a translation-free group, its name comes from church windows. There are infinitely many rosette groups which can be classified into two substantially different subgroups. One of them consists of groups of rotations about a single point: rotations with integer multiples of  $2\pi/n$ . They belong to the cyclic rosette group, written  $C_n$ . These groups do not have reflection symmetry. Besides the mentioned rotations, if the group also have  $n$  reflection symmetries with axes going through the centre of rotation, they belong to the dihedral group, written  $D_{2n}$ .

Rosettes GeoGebra: <https://www.geogebra.org/classic/ct2w6ayx>

**Prior knowledge:** Isometries

**Comments:**

For the 10-14 age group, Kali is the best program to use, for the 14-18 age group GeoGebra (maybe Kali as an introduction and then GeoGebra). Here you should try the slider too.

Internet searching will probably need some help, especially for the younger age group. Carefully chosen keywords should be tried, possibly in the foreign languages learnt.

**Connection to other subjects/topics/areas:**

Art, IT