Lesson 6: Using and interpreting a mathematical model

Goals

- Interpret a scatter plot and line of fit that model temperature and latitude, and explain (orally) limitations of the model.
- Use a mathematical model of bivariate data to make predictions (in writing).

Lesson Narrative

In the last in the sequence of three lessons, students analyse the model and use it to make a prediction and to draw conclusions. They interpret mathematical features of their model (gradient and intercepts of the line), and discuss limitations of the model.

Addressing

- Use functions to model relationships between quantities.
- Investigate patterns of association in bivariate data.

Instructional Routines

- Collect and Display

Student Learning Goals

Let's use a model to make some predictions.

6.1 Using a Mathematical Model

Optional: 15 minutes

In the previous activity, students drew a line that best fit the latitude-temperature data and found the equation of this line. The line is a mathematical model of the situation. In this lesson, they use their model to make predictions about temperatures in cities that were not included in the original data set. In the next activity, they also interpret the gradient of the line and the intercepts in the context of this situation. This leads to a discussion of the limitations of the mathematical model they developed.

Instructional Routines

- Collect and Display

Launch

Students in same groups of 3–4. If available, tell the students the latitude and average high temperature in September in their city.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organisational skills in problem solving. For example, present one question at a time and monitor students to

ensure they are making progress throughout the activity.
Supports accessibility for: Organisation; Attention

Student Task Statement

In the previous activity, you found the equation of a line to represent the association between latitude and temperature. This is a *mathematical model*.

1. Use your model to predict the average high temperature in September at the following cities that were not included in the original data set:

    a. Detroit (Lat: 42.14)

    b. Albuquerque (Lat: 35.2)

    c. Nome (Lat: 64.5)

    d. Your own city (if available)

2. Draw points that represent the predicted temperatures for each city on the scatter plot.

3. The actual average high temperature in September in these cities were:

    - Detroit: 74°F

    - Albuquerque: 82°F

    - Nome: 49°F

    - Your own city (if available):

    How well does you model predict the temperature? Compare the predicted and actual temperatures.

4. If you added the actual temperatures for these four cities to the scatter plot, would you move your line?

5. Are there any outliers in the data? What might be the explanation?

Student Response

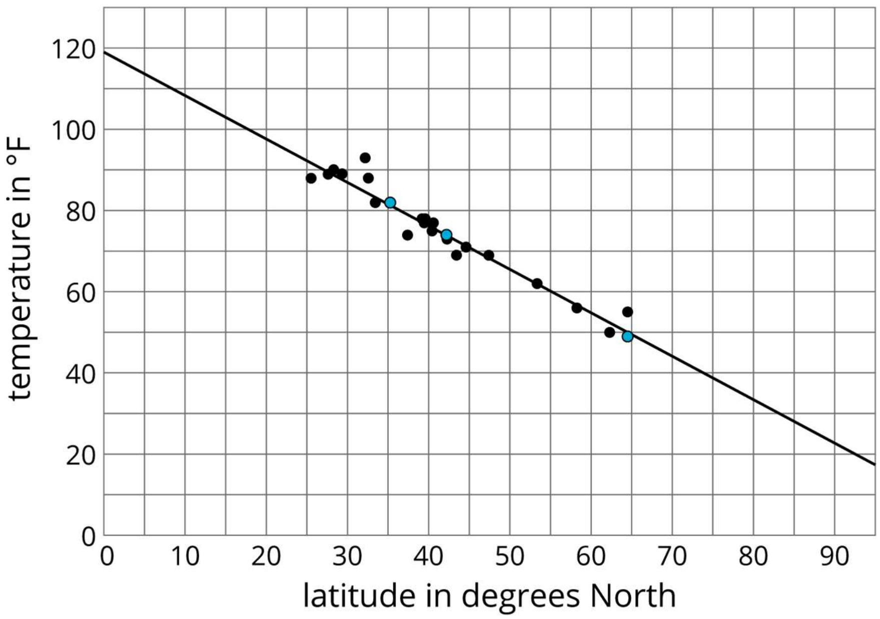
1. Answers may vary slightly depending on the equation of the line determined by eye.

    a. Detroit: actual 74°F, prediction: $119 - 1.07(42.14)$ or 73.9°F

    b. Albuquerque: actual 82°F, prediction: $119 - 1.07(35.2)$ or 81.3°F

    c. Nome: actual 49°F, prediction: $119 - 1.07(64.5)$ or 50°F

2.



3. The predictions are very close to the actual data.

4. They would not cause a noticeable change to the line.

5. There aren't big outliers in the data. The two points that lie the farthest from the line represent Tucson and Fairbanks. This might have geographical reasons—a desert or sea climate, respectively.

Activity Synthesis

Invite some groups to share their results and compare predictions for the different cities. Ask if students think that the line is a good mathematical model to predict the temperature in a location if you know the latitude.

Representing, Speaking, Listening: Collect and Display. Use this routine to collect verbal and written ideas about students' mathematical models and the limitations of them that they see. During the synthesis discussion, chart key ideas that students share out about the gradient, intercept, and differences in their predictions. Remind students that we call this a "mathematical model" (i.e., the line they drew). It is one way to look at data. Ask students, "What are the limitations of this model in making predictions?" Add their language to your chart. Reference this chart in the following lesson and make revisions as needed with student input.

Design Principle(s): Support sense-making; Maximise meta-awareness

6.2 Interpreting a Mathematical Model

Optional: 15 minutes

Students interpret the gradient and intercepts in the context of the situation. They also discuss limitations of the mathematical model.

This activity can be extended by having students investigate if temperature on other continents or across continents fits the same pattern that we found for North America.

Launch

Keep students in same groups of 3–4.

Anticipated Misconceptions

Students may want to say "For every one unit increase in x, y decreases by 1.07 units." Ask them to use the specific units and quantities in the model, latitude in degrees north and temperature in degrees Fahrenheit.

Student Task Statement

Refer to your equation for the line that models the association between latitude and temperature of the cities.

1. What does the gradient mean in the context of this situation?
2. Find the vertical and horizontal intercepts and interpret them in the context of the situation.
3. Can you think of a city or a location that could not be represented using this same model? Explain your thinking.

Student Response

1. For every degree latitude moving north, the temperature decreases by 1.07°F.
2. Vertical: The temperature at 0 degrees north (that is, on the equator) is 119°F. Horizontal: A latitude where high temp is 0°F would have to be over 100 degrees north, which doesn't exist.
3. Latitudes are restricted to 0–90 degrees by the situation. Factors other than latitude influence temperature, and those factors seem to be more important close to 0 degrees and close to 90 degrees. The model only used cities in North America. It should not be used to make predictions about temperatures on other continents without checking similar data there first.

Activity Synthesis

Invite students to share their responses. Discuss the limitations and uses of the model. Consider asking the following questions:

- "What are some limitations of the model?"
- "Do limitations mean that the model is not good?" (No, it just means that we have to be aware of when we can use it and when we can't use it. Our model is pretty good for latitudes between 25 and 65 degrees north, and for locations in North America.)

- “What questions do you have about predicting temperature?”
- “How could you extend your investigation of predicting temperature or the weather?”

Engagement: Develop Effort and Persistence. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class. *Supports accessibility for: Language; Social-emotional skills; Attention*