

Lesson 16: Solving more ratio problems

Goals

- Choose and create diagrams to help solve problems involving ratios and the total amount.
- Compare and contrast (orally) different representations of and solution methods for the same problem.

Learning Targets

- I can choose and create diagrams to help think through my solution.
- I can solve all kinds of problems about equivalent ratios.
- I can use diagrams to help someone else understand why my solution makes sense.

Lesson Narrative

In this lesson, students use all representations they have learned in this unit—double number lines, tables, and bar models—to solve ratio problems that involve the sum of the quantities in the ratio. They consider when each tool might be useful and preferable in a given situation and why. In so doing, they make sense of situations and representations, and are strategic in their choice of solution method.

Addressing

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, bar models, double number line diagrams, or equations.

Instructional Routines

- Information Gap Cards
- Three Reads
- Discussion Supports
- Poll the Class
- Think Pair Share

Required Materials

Graph paper

Pre-printed slips, cut from copies of the blackline master

<p>Salad Dressing and Moving Boxes Problem Card 1</p> <p>A recipe for salad dressing uses oil and vinegar. Clare made a certain amount of this dressing. How much oil did she use?</p>	<p>Salad Dressing and Moving Boxes Data Card 1</p> <p>The recipe calls for 4 parts oil.</p> <ul style="list-style-type: none"> • The recipe calls for 3 parts vinegar. • The ratio of oil to vinegar in the recipe is 4 : 3. • The ratio of vinegar to oil in the recipe is 3 : 4. • Clare made a total of 28 teaspoons of dressing.
<p>Salad Dressing and Moving Boxes Problem Card 2</p> <p>Andre and Han are moving boxes, each working at a constant rate. How long will it take Andre and Han to move all the boxes?</p>	<p>Salad Dressing and Moving Boxes Data Card 2</p> <ul style="list-style-type: none"> • Andre can move 4 boxes every half hour. • Han can move 5 boxes every half hour. • The ratio of boxes moved by Andre to boxes moved by Han is 4 : 5. • The ratio of boxes moved by Han to boxes moved by Andre is 5 : 4. • There are 72 boxes that need to be moved.

Rulers

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

You will need the Solving More Ratio Problems Info Gap Blackline Master for this activity. Make one copy for every 6 students, and cut them up ahead of time. It is recommended to use a different colour paper for each of the four pages, to make them easier to organise. A class set could be reused if you have more than one class.

Student Learning Goals

Let's compare all our strategies for solving ratio problems.

16.1 You Tell the Story

Warm Up: 10 minutes

This warm-up reminds students of previous work with bar models and encourages a different way to reason with them. Students are given only a bar model and are asked to generate a concrete context to go with the representation.

Students' stories should have the following components:

-
- the same unit for both quantities in the ratio
 - a ratio of 7 : 3
 - scaling by 3, or 3 units per part
 - They may also have a quantity of 30 that represents the sum of the two quantities in the ratio.

As students work, identify a few different students whose stories are clearly described and are consistent with the diagram so that they can share later.

Instructional Routines

- Think Pair Share

Launch

Ask students to share a few things they remember about bar models from the previous lesson. Students may recall that:

- We draw one bar for every quantity in the ratio. Each bar has parts that are the same size.
- We draw as many parts as the numbers in the ratio show (e.g., a 2 : 3 ratio: we draw 2 parts in a bar and 3 parts in another bar).
- Each part represents the same value.
- Bar models can be used to think about a ratio of parts and the total amount.

Tell students their job is to come up with a valid situation to match a given bar model. Give students some quiet thinking time, and then time to share their response with a partner.

Anticipated Misconceptions

Students may misunderstand the meaning of the phrase “with two quantities” and simply come up with a situation involving ten identical groups of three. Point out that the phrase means that the row of seven groups of three should represent something different than the row of three groups of three.

Students may also come up with a situation involving different units, for example, quantity purchased and cost, or distance travelled and time elapsed. Remind them that the parts of bar are meant to represent the same value, so we need a situation that uses the same units for both parts of the ratio.

Student Task Statement

Describe a situation with two quantities that this bar model could represent.

3	3	3	3	3	3	3
---	---	---	---	---	---	---

3	3	3
---	---	---

Student Response

Answers vary. Examples might be

- One batch of purple paint is made by mixing 7 cups of blue with 3 cups of red. In three batches, there are 30 cups of purple paint, which is made of 21 cups of blue paint and 9 cups of red paint.
- There are 30 fish in an aquarium. The ratio of blue fish to red fish is 7 : 3. There are 21 blue fish and 9 red fish.

Activity Synthesis

Invite a few students to share their stories with the class. As they share, consider recording key details about each story for all students to see. Then, ask students to notice similarities in the different scenarios.

Guide students to see that they all involve the same units for both quantities of the ratio, a ratio of 7 to 3, and either 3 units per part or scaling by 3. They may also involve an amount of 30 units, representing the sum of the two quantities.

16.2 A Trip to the Aquarium

20 minutes

This task prompts students to solve a single problem using a triple number line, a table, or a bar model. Either assign each student a representation or allow students to choose the representation they prefer. During the following discussion, they will compare and contrast the three representations and identify the relative merits of the different representations for the different problems.

Students will have varying opinions about which representation they prefer and why. Their views may stem from observations such as:

- Number lines and tables involve scaling up individual quantities to find the total. Bar models involve starting with the total and breaking it down into equal groups.
- It is hard to take shortcuts with number line diagrams.
- We can use the number line diagram or the table efficiently by thinking, “17 times what is 85?” This value tells us how many “batches” of tickets the teacher had to order.
- When using a bar model, it was easier to count 17 groups and compute $85 \div 17$ to find how many tickets each group needs.

Students may struggle to represent the problem with a bar model. As they work, notice any trends that may need to be addressed with the class.

Instructional Routines

- Three Reads
- Poll the Class

Launch

Tell students that they will now solve a ratio problem in one of three different ways. Either assign each student one of the representations or instruct them to choose one representation to use.

Give students quiet think time to complete the activity and then, optionally, time to share their responses with a small group or in pairs.

Representation: Develop Language and Symbols. Maintain a display of important terms and vocabulary. During the launch, take time to review the following terms from previous lessons that students will need to access for this activity: bar models, quantities, ratios, batch.

Supports accessibility for: Memory; Language

Reading: Three Reads. In this routine, students are supported in reading a mathematical text, situation, or word problem three times, each with a particular focus. During the first read, students focus on comprehending the situation; during the second read, students identify quantities; during the third read, students brainstorm possible strategies to answer the question. The question to be answered does not become a focus until the third read so that students can make sense of the whole context before rushing to a solution. The purpose of this routine is to support students' reading comprehension as they make sense of mathematical situations and information through conversation with a partner.

Design Principle(s): Support sense-making

How It Happens:

1. In the first read, students read the problem with the goal of comprehending the situation. Do not reveal the questions with the three options for solving at this point.

Invite a student to read the description of the situation aloud while everyone else reads with them and then ask, "What is this situation about?"

Allow one minute to discuss with a partner, and then share with the whole class. A clear response would be: "buying tickets for a class trip to an aquarium."

2. In the second read, students analyse the mathematical structure of the story by naming quantities.

Invite students to read the problem aloud with their partner or select a different student to read to the class and then prompt students by asking, "What can be counted

or measured in this situation? For now we don't need to focus on how many or how much of anything, but what can we count in this situation?" Give students one minute of quiet think time followed by another minute to share with their partner. Quantities may include: total number of tickets bought; number of students; number of chaperones; ratio of chaperones to students.

3. In the third read, students brainstorm possible mathematical strategies to answer the question, "How many tickets are for chaperones, and how many are for students?" (Still do not reveal the three given options for solving.)

Invite students to read the problem aloud with their partner or select a different student to read to the class. Instruct students to think of ways to approach the question without actually solving the problem.

Consider using these questions to prompt students: "What strategy or method would you try first?," "How could a diagram help you approach this question?," and "Can you think of a different way to solve it?"

Give students one minute of quiet think time followed by another minute to discuss with their partner. Provide this sentence frame as partners discuss: "One way to approach the question would be to...."

Sample responses include: "I would figure out possibilities for the number of students and number of chaperones and see which one adds up to 85.", "I know that if I divide 85 by 2, it won't work because there are more students than chaperones, so the groups are not equal, so I know I have to split it up differently.", "I would draw a bar model to figure out different lengths that fit the ratio of students to chaperones.", and "I would use a double number line/table to scale up the ratio of students to chaperones." This will help students concentrate on making sense of the situation before rushing to a solution or method.

4. As partners are discussing their strategies, select 1–2 students to share their ideas with the whole class.

Listen for quantities that were mentioned during the second read, and take note of strategies that make explicit the relationships between number of tickets and number of students, number of tickets and number of chaperones, or number of students and number of chaperones. As students are presenting their strategies to the whole class, create a display that summarises ideas about the question.

5. Post the display where all students can use it as a reference, and finally reveal the actual problems and ensure that a variety of strategies are chosen by students.

Anticipated Misconceptions

The number line and table representations are organised similarly. For example, one could make progress with both of them simply by skip counting and keeping an eye out for a total of 85 people. The bar model, though, is organised in a much different way. Equivalent ratios

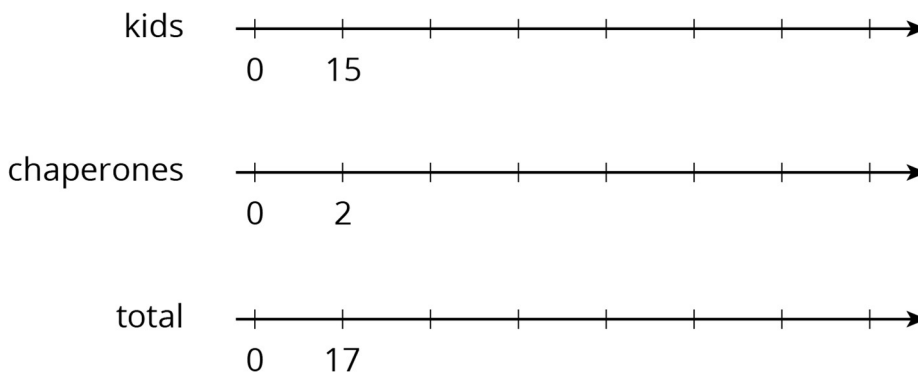
are not listed out, but rather equivalent ratios arise from thinking about how the diagram could represent any number of batches. Students may thus mistakenly treat the bar model like a double number line diagram—they may start writing 15, 30, 45, etc. in the “kids” bar, for example. Once this plays out, students *may* self-regulate once they notice there are only 2 boxes in the chaperones’ row. But they may decide to just draw more boxes! Reorient these students by asking how many parts of bar there are (17), and reminding them each part of bar represents equal numbers of people, and that there are 85 total people. The presentation of correct work during the discussion could be used as an opportunity to remediate, as well. For example, consider asking a student to explain what they understand about another student’s correct work.

Student Task Statement

Consider the problem: A teacher is planning a class trip to the aquarium. The aquarium requires 2 chaperones for every 15 students. The teacher plans accordingly and orders a total of 85 tickets. How many tickets are for chaperones, and how many are for students?

1. Solve this problem in *one* of three ways:

Use a triple number line.

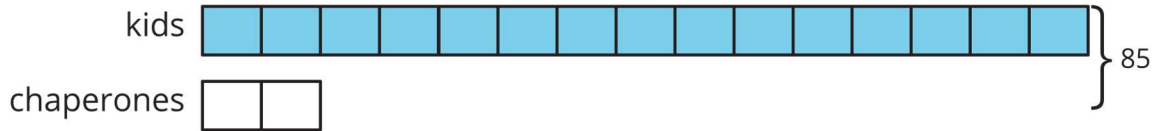


Use a table.

(Fill rows as needed.)

kids	chaperones	total
15	2	17

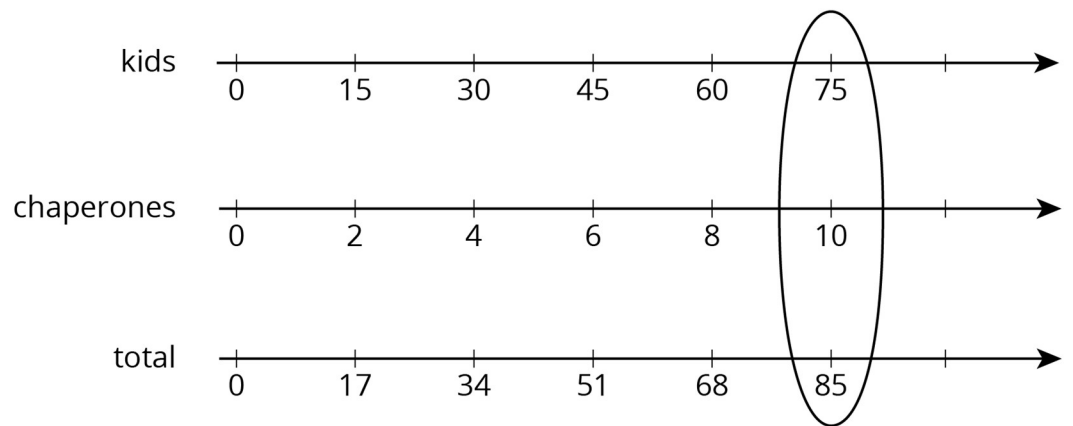
Use a bar model.



2. After your class discusses all three strategies, which do you prefer for this problem and why?

Student Response

1.

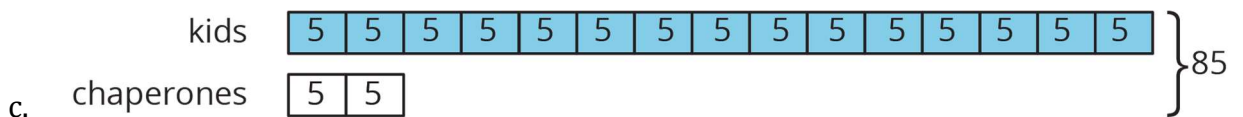


a.

kids	chaperones	total
15	2	17
30	4	34
45	6	51
60	8	68
75	10	85

} not enough

b.



c.

There are a total of 17 boxes. $85 \div 17 = 5$. Each part of bar is worth 5 tickets, so we can see that the teacher ordered 75 tickets for kids and 10 tickets for chaperones.

2. Answers vary.

Are You Ready for More?

Use the digits 1 through 9 to create three equivalent ratios. Use each digit only one time.

: is equivalent to : and :

Student Response

One solution is $6 : 3$, $18 : 9$, and $54 : 27$.

Note: this problem was inspired by the problem *Finding Equivalent Ratios* by Graham Fletcher published at openmiddle.com and used here with permission.

Activity Synthesis

Before debriefing as a class, consider arranging students in groups of 3, where a group includes one student who used each representation. Give them time to see if they got the same answer and compare and contrast the representations.

Solicit students' reactions to the three strategies, encouraging them to identify what is similar and different about the approaches. Consider polling the class for their preferred strategy. Ask 1–2 students favouring each method to explain why. This can also be a discussion about what worked well in this or that approach, and what might make this or that approach more complete or easy to understand. While there is no right or wrong answer with regards to their preferred strategy, look out for unsupported reasoning or misunderstandings.

Some students may prefer the bar model because the solution path seems more direct, but caution them against trying to use a bar model any time they see a ratio problem. Because bar models involve equal-sized parts, they can only be used to represent quantities with the same unit. If different units are involved, the parts of one bar and those of the other will not represent an identical quantity.

16.3 Salad Dressing and Moving Boxes

10 minutes

This activity gives students a chance to apply the different strategies they have learned to solve ratio problems and to decide on methods that would make the most sense in given situations. *If desired*, you can have students complete this activity as an info gap, by instructing them to close their books or devices and distributing slips cut from the blackline master. Alternatively, students can simply complete the two problems that are printed in the task statement.

Each problem may lend itself better to a particular representation than to others. Below are sample arguments in support of a representation for each problem, though options may vary:

- The salad dressing problem can be well represented by all three representations because it involves small numbers and simple multiplication.
- The box-moving problem would be inefficient to represent with a number line diagram, since it would require making 8 half-hour jumps to find the total of 72. Either a bar model or a table showing more straightforward multiplication would be more efficient.

The time needed to solve the problems may depend on the representation students choose to use.

Instructional Routines

- Information Gap Cards
- Discussion Supports

Launch

Tell students they will continue to practice using the tools at their disposal—number lines, tables, and bar models—to solve ratio problems.

Provide access to graph paper and rulers. Arrange students in groups of 2.

If using the info gap routine: Explain the info gap structure and consider demonstrating the protocol. In each group, distribute a problem card to one student and a data card to the other student. Give students who finish early a different pair of cards and ask them to switch roles.

If not using the info gap routine: Ask students to complete both questions and then compare their strategies with their partner. Partners should work together to resolve any discrepancies.

Action and Expression: Internalise Executive Functions. Begin with a small-group or whole-class demonstration and think aloud of a sample situation to remind students how to use the info gap structure. Keep the worked-out table and double number lines on display for students to reference as they work.

Supports accessibility for: Memory; Conceptual processing *Conversing: Information Gap.* Use this routine to give students a purpose for discussing information necessary to solve ratio problems. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?”

Design Principle(s): Cultivate Conversation

Anticipated Misconceptions

While thinking through problems, it is common for students to hold the meanings of their representations (numbers, quantities, markings, etc.) in their heads without writing them down. When students are getting their solutions ready for others to look at, though, remind them of the importance of labelling quantities and units of measure and making the steps in their thinking clear.

Some students may choose the same strategy or representation each time. If their answers are accurate, this is fine. However, if time allows, ask them to check if they can verify their answer using an alternative strategy.

If using the info gap routine: Students holding a Problem Card may have trouble thinking of appropriate questions to ask their partner. Encourage them to revisit the problem at hand and think about the kinds of information that might be helpful or relevant. (For example, if the problem is about how long it would take to perform something, ask students how they usually gauge the amount of time needed for something. Ask, “What would the amount of time depend on?”)

Student Task Statement

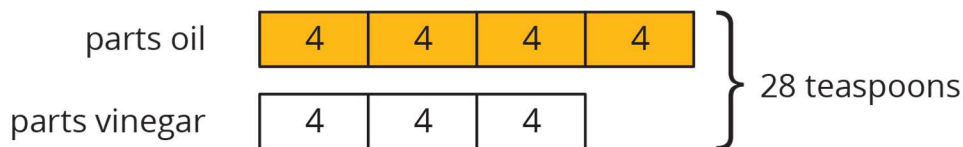
Solve each problem, and show your thinking. Organise it so it can be followed by others. If you get stuck, consider drawing a double number line, table, or bar model.

1. A recipe for salad dressing calls for 4 parts oil for every 3 parts vinegar. How much oil should you use to make a total of 28 teaspoons of dressing?
2. Andre and Han are moving boxes. Andre can move 4 boxes every half hour. Han can move 5 boxes every half hour. How long will it take Andre and Han to move all 72 boxes?

Student Response

Each problem can be approached with all three representations. The strategies shown reflect a straightforward or efficient approach for each problem (although this is a matter of opinion).

1. 16 teaspoons. $28 \div 7 = 4$ and $4 \times 4 = 16$.



2. 4 hours. The total number of boxes carried each half hour is 9. $72 = 9 \times 8$. Therefore, it takes $8 \frac{1}{2}$ -hour increments, or 4 hours, to move 72 boxes.

boxes moved by Andre	boxes moved by Noah	total boxes moved	elapsed time (hours)
4	5	9	$\frac{1}{2}$
32	40	72	4

Activity Synthesis

After students have had time to work, share the correct answers and ask students for their preferences about representations. Some guiding questions:

- “Is there a particular representation you tend to try first?”
- “Does one seem more efficient than the others?”
- “Did the situation in a problem affect your choice? If so, what features of a problem might steer you toward or away from a particular strategy?”

Speaking: Discussion Supports. Revoice student ideas to demonstrate mathematical language. Press for details in students’ explanations by requesting that students challenge an idea, elaborate on an idea, or give an example. This will help students communicate with more precise language.

Design Principle(s): Cultivate conversation

Lesson Synthesis

This lesson was all about understanding that there are different, valid representations to use for problems involving equivalent ratios. For some problems, one representation is easier to use than others. There are no hard and fast rules. As long as the diagram correctly shows the mathematics, and the problem solver can explain it, it’s okay. Some guidelines to draw out are:

- Bar models are most likely to be helpful when the parts of the ratio have the same kind of units and the sum of the quantities is meaningful in the context. For example, cups to cups; miles to miles; boxes moved to boxes moved.
- Number lines are a good choice when it helps to visualise how far apart numbers are from each other. They are harder to use with very big or very small numbers.
- Tables work well in almost all situations. Ask students to articulate good habits when solving equivalent ratio problems with the different representations. Some ideas might include:
 - Label each part of the diagram with what it represents.
 - Use brackets to indicate total amounts.
 - Make sure you read what the question is asking and answer it.
 - Make sure you make the answer easy to find.

- Include units in your answer. For example, instead of just writing “4,” write “4 cups.”

16.4 Pizza-making Party

Cool Down: 5 minutes

Student Task Statement

You are having a pizza-making party for a number of people. You will need 6 ounces of dough and 4 ounces of sauce for each person at this party. If you used a total of 130 ounces of dough and sauce all together,

1. How many ounces of dough were used at the party?
2. How many ounces of sauce were used at the party?
3. How many guests were at the party?

Student Response

1. 78 ounces of dough.
2. 52 ounces of sauce.

Possible strategy:

dough (ounces)	sauce (ounces)	total (ounces)
6	4	10
78	52	130

3. 12 guests. You bought ingredients for 13 people. $6 \times 13 = 78$ and $4 \times 13 = 52$. Since you bought some for yourself, there are 12 guests coming.

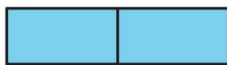
Student Lesson Summary

When solving a problem involving equivalent ratios, it is often helpful to use a diagram. Any diagram is fine as long as it correctly shows the mathematics and you can explain it.

Let’s compare three different ways to solve the same problem: The ratio of adults to kids in a school is 2 : 7. If there is a total of 180 people, how many of them are adults?

- *Bar models* are especially useful for this type of problem because both parts of the ratio have the same units (“number of people”) and we can see the total number of parts.

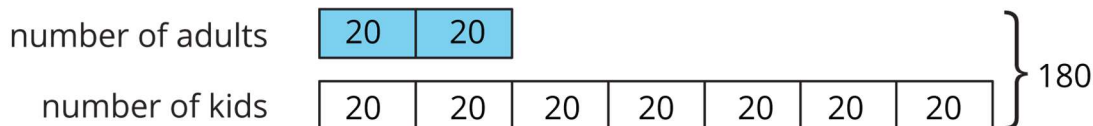
number of adults



number of kids

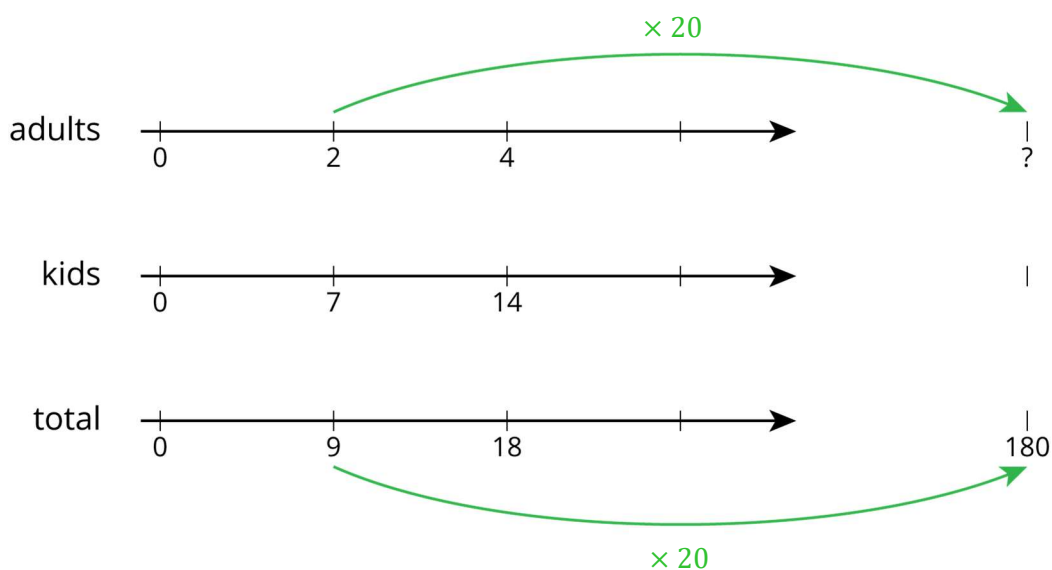


This bar model has 9 equal parts, and they need to represent 180 people total. That means each part represents $180 \div 9$, or 20 people.



Two parts of the bar model represent adults. There are 40 adults in the school because $2 \times 20 = 40$.

- *Double or triple number lines* are useful when we want to see how far apart the numbers are from one another. They are harder to use with very big or very small numbers, but they could support our reasoning.



- *Tables* are especially useful when the problem has very large or very small numbers.

	adults	kids	total
	2	7	9
	?		180

Green arrows point from the question mark in the 'adults' column to the '2' in the row above, and from the '180' in the 'total' column to the '9' in the row above, both labeled 'x 20'.

We ask ourselves, “9 times what is 180?” The answer is 20. Next, we multiply 2 by 20 to get the total number of adults in the school.

Another reason to make diagrams is to communicate our thinking to others. Here are some good habits when making diagrams:

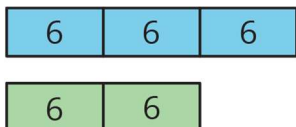
- Label each part of the diagram with what it represents.
- Label important amounts.
- Make sure you read what the question is asking and answer it.

- Make sure you make the answer easy to find.
- Include units in your answer. For example, write “4 cups” instead of just “4.”
- Double check that your ratio language is correct and matches your diagram.

Lesson 16 Practice Problems

Problem 1 Statement

Describe a situation that could be represented with this bar model.



Solution

Answers vary. Sample response: There are 30 people at a movie. The ratio of teenagers to adults is 3 to 2. There are 18 teenagers and 12 adults.

Problem 2 Statement

There are some nickels, dimes, and quarters in a large piggy bank. For every 2 nickels there are 3 dimes. For every 2 dimes there are 5 quarters. There are 500 coins total.

- How many nickels, dimes, and quarters are in the piggy bank? Explain your reasoning.
- How much are the coins in the piggy bank worth? (A nickel is worth 5 cents, a dime is worth 10 cents and a quarter is worth 25 cents.)

Solution

- 80 nickels, 120 dimes, 300 quarters. Possible strategies:
 - For every 2 nickels there are 3 dimes, so for every 4 nickels there are 6 dimes. For every 2 dimes there are 5 quarters, so for every 6 dimes there are 15 quarters. The ratio of nickels to dimes to quarters is 4 to 6 to 15, a total of 25 coins in the group. There are 500 coins, which means 20 groups of coins, since $500 \div 25 = 20$. There are 80 nickels ($20 \times 4 = 80$), 120 dimes ($20 \times 6 = 120$), and 300 quarters ($20 \times 15 = 300$).
 - Using a table:

nickles	dimes	quarters	total
4	6	15	25
16	24	60	100
80	120	300	500

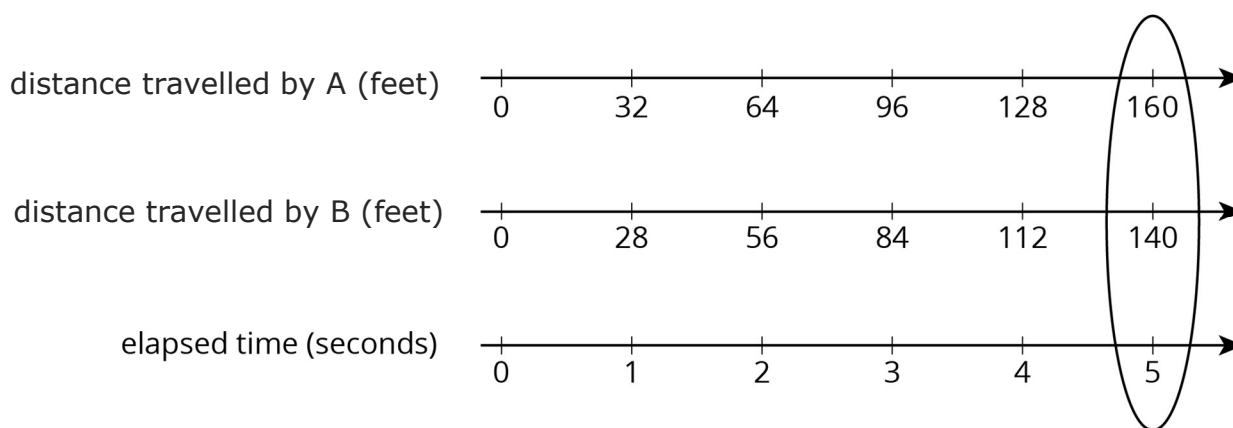
- b. \$91. The nickels are worth \$4, the dimes are worth \$12, and the quarters are worth \$75, making a total of \$91.

Problem 3 Statement

Two horses start a race at the same time. Horse A gallops at a steady rate of 32 feet per second and horse B gallops at a steady rate of 28 feet per second. After 5 seconds, how much farther will horse A have travelled? Explain or show your reasoning.

Solution

Horse A will have travelled 20 feet farther. Possible reasoning:



$$160 - 140 = 20$$

Problem 4 Statement

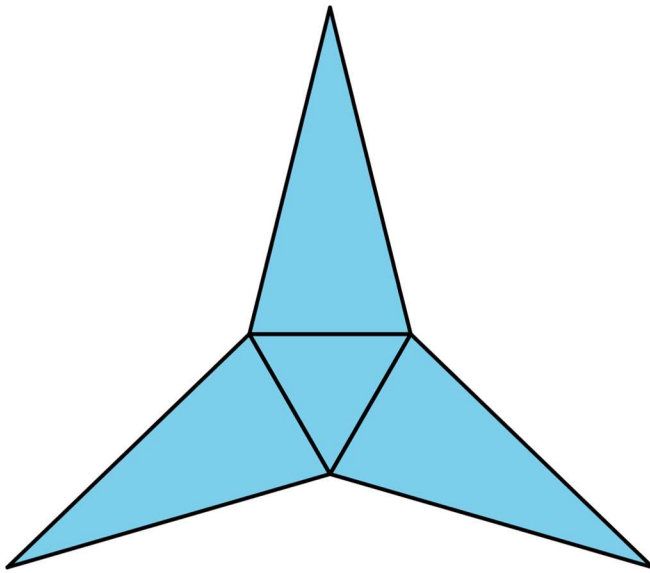
Andre paid £13 for 3 books. Diego bought 12 books priced at the same rate. How much did Diego pay for the 12 books? Explain your reasoning.

Solution

£52. Sample explanation: 12 is 4 groups of 3 so Diego’s books will cost 4 times as much as Andre’s and $4 \times 13 = 52$.

Problem 5 Statement

Which polyhedron can be assembled from this net?

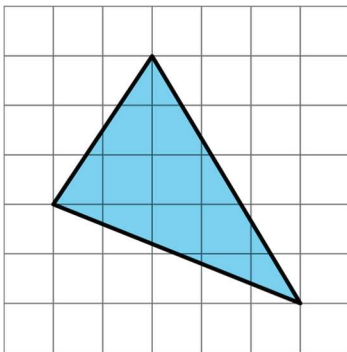


- a. A triangle-based pyramid
- b. A trapezium -shaped prism
- c. A rectangle-based pyramid
- d. A triangular prism

Solution A

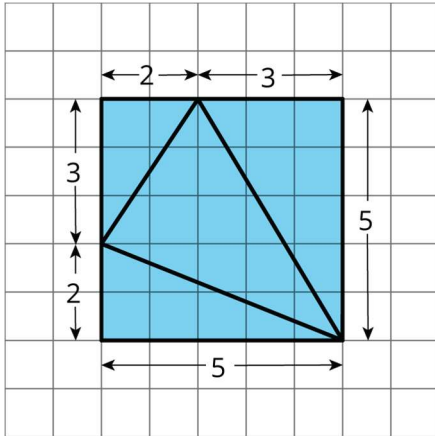
Problem 6 Statement

Find the area of the triangle. Show your reasoning. If you get stuck, consider drawing a rectangle around the triangle.



Solution

9.5 square units. Explanations vary. Sample response:



Surround the triangle with a 5 by 5 unit square, which has an area of 25 square units. From the area of the square, subtract the areas of the three right triangles. The total area of the right triangles is 15.5 square units, because $3 + 5 + 7.5 = 15.5$. The area of the given triangle is 9.5 square units, since $25 - 15.5 = 9.5$.



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