Lesson 4: How do we choose?

Goals

- Apply reasoning about ratios and percentages to analyse (orally and in writing) voting situations involving two choices.
- Comprehend the term "majority" (in spoken and written language).
- Critique (using words and other representations) a statement reporting the results of a vote.

Lesson Narrative

This lesson is optional. This is the first of three lessons that explore the mathematics of voting: democratic processes for making decisions. The activities in these lesson build on each other. Doing all of the activities in the three lessons would take more than three lessons—possibly as many as five. It is not necessary to do the entire set of activities to get some benefit from them, although more connections are made the further one gets. As with all lessons in this unit, all related topics have been addressed in prior units; this lesson provides an optional opportunity to go more deeply and make connections between concepts.

The activities in this lesson are about voting on issues where there are two choices. Students use equivalent ratio concepts and skills developed in year 7 to compare voting results of two groups, to determine whether an issue wins an election with a majority rule, and discover that a few people can determine the results of an election when very few people vote.

Most of the activities use students' skills from earlier units to reason about ratios in the context of real-world problems. While some of the activities do not involve much computation, they all require serious thinking. In many activities, students have to make choices about how to assign votes and justify their methods.

Most importantly, this lesson addresses topics that are important for citizens in a democracy to understand. Teachers may wish to collaborate with a civics or politics teacher to learn how the fictional KS3 situations in this lesson relate to real-world elections.

Addressing

- Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2 : 1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."
- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, bar models, double number line diagrams, or equations.

- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percentage.

Instructional Routines

- Collect and Display
- Three Reads
- Compare and Connect

Required Materials

Coloured pencils
Four-function calculators
Graph paper
Scissors

Student Learning Goals

Let's vote and choose a winner!

4.1 Which Was "Yessier"?

Optional: 10 minutes

This activity gives students a chance to recall and use various ratio strategies in the context of a voting problem. Two classes voted on a yes or no question. Both classes voted yes. Students are asked to determine which class was more in favour ("yessier"). Students need to make sense of the invented word "yessier" by thinking about how it might be quantified.

Monitor for students who use different ways to make sense of the problem.

Instructional Routines

- Collect and Display

Launch

Arrange students in groups of 2–4. Ask students to use the mathematical tools they know to answer the question.

Anticipated Misconceptions

Students may not understand the question. The word "yessy" was invented by other students solving a similar problem. It is not standard English.

If students do not understand what comparing the two classes means, give a more extreme example, such as comparing 20 to 1 with 11 to 10. The 20 to 1 class is much more yessy because almost everyone said yes. The other class had almost equal yesses and nos.

Student Task Statement

Two year 7 classes, A and B, voted on whether to give the answers to their maths problems in poetry. The "yes" choice was more popular in both classes.

	yes	no
class A	24	16
class B	18	9

Was one class more in favour of maths poetry, or were they equally in favour? Find three or more ways to answer the question.

Student Response

Answers vary. Sample response:

One strategy is to find simpler equivalent ratios, then compare them by finding a common amount.

Class B's ratio was 2 : 1, since 18 is two groups of 9. This means there were twice as many yesses as nos. If class A was the same "yessiness" as class B, it would have 24 yesses and 12 nos. There are more than 12 nos, so that means that class A is more "no-y," which is less "yessy." Alternatively, equally "yessy" could be 32 yesses and 16 nos. There are fewer than 32 yesses, so class A is less "yessy" than class B.

The same approach can work using the ratio 3 : 2 for class A. A 3 : 2 ratio with 18 yesses would have 12 nos, more than the 9 nos in class B.

Here are some equivalent ratios shown in a table for each class. Many others are possible.

	yes	no
class A	24	16
	12	8
	3	2
	6	4
	18	12
	yes	no
class B	18	9
	6	3
	2	1

	4	2
	24	12

Another strategy involves finding the totals, then comparing percentages. The ratios could also be shown on double number lines, one pair for each class.

	yes	no	total	fraction yesses	percentage yesses
class A	24	16	40	$\frac{24}{40}=\frac{3}{5}$	60%
class B	18	9	27	$\frac{18}{27}=\frac{2}{3}$	66.7%

Activity Synthesis

The situation is mathematically the same as other rate comparison problems, such as comparing the tastes or colours of two mixtures.

Invite several students to present different methods, at least one who used a ratio of yesses to nos, and another who used a ratio of yesses to all students. Make sure to present a solution using percentages.

If more than a few students did not use multiplicative techniques (for example, if they compared only the yesses, or subtracted to find how many more yesses than nos) remind them of the rate comparisons they did earlier in the year, in which they could check by tasting a drink mixture or looking at the colour of a paint mixture.

Speaking, Listening, Representing: Collect and Display. Use this routine to display examples of the different methods students used to determine which class was “yessier." Circulate and collect examples of formal and informal language, calculations, tables, or other writting. Ask students to compare and contrast percentage calculations with fractions and equivalent ratios.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

4.2 Which Class Voted Purpler?

Optional: 10 minutes

This activity is the same type of situation as the previous one: comparing the voting of two groups on a yes or no issue. However, the numbers make it more difficult to use "part to part" ratios. Again, students need to be thinking about how to make sense of and quantify the class voting decisions.

Launch

Arrange students in groups of 2–4. Provide access to four-function calculators.

Anticipated Misconceptions

If students do not understand what comparing the two classes means, give a more extreme example, such as comparing 20 to 1 with 11 to 10. The 20 to 1 class is much more purple because almost everyone said yes. The other class had almost equal yesses and nos.

Students may be stuck with the difficult-looking numbers, expecting to be able to do calculations to create equivalent ratios mentally. Suggest that they find the total number of votes in each class.

Students may calculate percentages incorrectly, forgetting that percentages are rates out of 100. So it is *not* correct to say that room A has 54% yesses. However, you can carefully make sense of this percentage as a comparison: class A had 54% as many nos as yesses; class B had 61% as many nos as yesses. This means that class B was more no-y, so it was less yessy than class A.

Student Task Statement

The school will be painted over the summer. Students get to vote on whether to change the colour to purple (a "yes" vote), or keep it a beige colour (a "no" vote).

The headteacher of the school decided to analyse voting results by class. The table shows some results.

In both classes, a majority voted for changing the paint colour to purple. Which class was more in favour of changing?

	yes	no
class A	26	14
class B	31	19

Student Response

Answers vary, Possible response:

Room A was "yessier," with 65% yesses. Room B had a smaller percentage of yesses, 62%.

	yes	no	total	fraction yesses	percentage yesses
room A	26	14	40	$\frac{26}{40} = \frac{13}{20}$	65%
room B	31	19	50	$\frac{31}{50}$	62%

Ratios could be used, but calculations are more difficult. Here are some equivalent ratios that could be used. Lines marked with a * have a common number of nos. The line with a ** has a common number of yesses.

	yes	no
room A	26	14
	13	7
*	$13 \times 19 = 247$	$7 \times 19 = 133$
*	$26 \times 19 = 494$	$14 \times 19 = 266$
**	$13 \times 31 = 403$	$7 \times 31 = 217$
	yes	no
room B	31	19
*	$31 \times 7 = 217$	$19 \times 7 = 133$
*	$31 \times 14 = 434$	$19 \times 14 = 266$
**	$31 \times 13 = 403$	$19 \times 13 = 247$

A rate of nos per yes also makes sense: room A has about 0.54 nos per yes, and room B has about 0.61 nos per yes. This makes room B more no-y, which means less yessy. Unit rate for each ratio: room A has about 1.9 yesses per no, while room B has about 1.6 yesses per no. Therefore, room A is yessier.

Activity Synthesis

Invite students to show different ways of solving the problem, including using equivalent ratios and percentages. Ask students to explain their thinking. Correct ideas with incorrect calculations are still worth sharing.

Representation: Develop Language and Symbols. Display or provide charts with symbols and meanings. Once students have determined which class was more in favour of changing, pause the class. Invite students to share their strategies for finding each fraction using percentages and equivalent ratios to justify their reasoning. Create a display that includes each strategy labelled with the name and the fraction they represent of a total. Keep this display visible as students move on to the next problems.
Supports accessibility for: Conceptual processing; Memory

4.3 Majorities

Optional: 10 minutes

This activity introduces the idea of requiring a majority. A majority is a voting rule that is used for issues where it is important to have more than just barely above half of the voters agreeing. To win, a choice must have more than the given fraction of the votes. In this activity, two majority rules are given: one as a fraction, one as a percentage. Students find a fraction of the total votes and a percentage of the total votes. They then compare the fraction and the percentage.

This activity encourages students to think about votes in ratios and percentages. Students make sense of problems and reason quantitatively.

Instructional Routines

- Compare and Connect

Launch

Arrange students in groups of 2–4. Provide access to a four-function calculator.

Explain what a majority is: "In many voting situations, a choice that wins a *majority* of the votes wins. A majority is more than half the votes. So if 1 000 votes were cast, a majority is any number over 500; 501 is the smallest number of votes that can win.

Many groups have special election rules for very important issues. Sometimes they require a larger majority: to win, you need more than a certain fraction that is more than half. For raising taxes, some governments require a $\frac{2}{3}$ majority. To change (amend) the U.S. constitution, an amendment must get a $\frac{2}{3}$ majority of both the Senate and House of Representatives, and be ratified by $\frac{3}{4}$ of the states. Sometimes majorities are described as percentages, such as 60%."

Representation: Internalise Comprehension. Activate or supply background knowledge about reasoning quantitatively about operations involving decimals and percentages. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

1. Another school is also voting on whether to change their school's colour to purple. Their rules require a $\frac{2}{3}$ majority to change the colours. A total of 240 people voted, and 153 voted to change to purple. Were there enough votes to make the change?

2. This school also is thinking of changing their mascot to an armadillo. To change mascots, a 55% majority is needed. How many of the 240 students need to vote "yes" for the mascot to change?

3. At this school, which requires more votes to pass: a change of mascot or a change of colour?

Student Response

1. 153 votes are not enough to win.
   One method: $\frac{2}{3}$ of 240 is twice as much as $\frac{1}{3}$ of 240. $\frac{1}{3}$ of 240 is 80, so $\frac{2}{3}$ of 240 is 160.
   Since you need one vote more than $\frac{2}{3}$ of 240, 161 votes are needed to win.

A method with more calculation: $\frac{153}{240}$ of the votes were for changing the colour. We need to check that this fraction is greater than $\frac{2}{3}$. Long division or a calculator would do this, by converting to a decimal: $\frac{153}{240} \approx 0.64$, and $\frac{2}{3} \approx 0.67$. So 153 is not enough votes to change.

2. 133 votes are needed to change the mascot. 55% of 240 is $\frac{55}{100} \times 240$. This can be computed with decimal multiplication: $(0.55)(240) = 132$ or with fraction multiplication: $\frac{55}{100} \times 240 = \frac{11}{20} \times 240 = 11 \times \frac{240}{20} = 11 \times 12 = 132$. Since a majority is more votes than the fraction of the total, 133 votes are needed to change the mascot.

3. A $\frac{2}{3}$ majority needs more votes than a 55% majority. The question is asking: which is more, $\frac{2}{3}$ of a number, or 55% of the same number? Since we don't know what the number is in advance, we can compare the fractions $\frac{2}{3}$ and $55\% = \frac{55}{100}$. Writing both as decimals, we see that $\frac{2}{3} \approx 0.67$ and $55\% = 0.55$, so it takes more votes to change the colour. This question can also be answered by comparing fractions using common denominators or common numerators. Here is one way: $\frac{2}{3} = \frac{200}{300}$ and $\frac{55}{100} = \frac{165}{300}$.

   Reasoning about fractions can give a quicker answer: since $\frac{2}{3}$ of 240 is more than 55% of 240, $\frac{2}{3}$ of any number is more than 55% of the same number, 133.

Activity Synthesis

Choose one or more students' methods to present.

Conversing, Speaking, Listening: Compare and Connect. Before the whole-class discussion, invite students to share their work with a partner or small group. Display the following questions to support conversation: "What was your strategy?", "Does anyone want to add on to _____'s strategy?", "Did anyone solve the problem in a similar way, but would explain it differently?", and "Does anyone have a completely different method for solving?" Listen for and amplify observations that include mathematical language and reasoning.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

4.4 Best Restaurant

Optional: 20 minutes

This activity shows how a few people can make a decision if many people don't vote. The mathematics involves repeated "percentage of" operations, each percentage giving a smaller amount than the previous step. The main issue in this problem is to identify "percentage of what?" for each percentage. The first percentage is 25% of the people in town subscribe to the newspaper. The second percentage is 20% of the result of the previous number, and the third is 80% of the second result.

Students need to give a written explanation, clearly show their calculations by writing expressions and equations, and make a diagram that accurately shows the sizes of all the groups in the problem. The diagram might be on a 10 × 10 grid, or a bar model. Graph paper is a good way to make sure the sizes are right. Bar models can also be made with a folded strip of paper, if students are accustomed to folding fractions.

Instructional Routines

- Three Reads

Launch

Arrange students in groups of 2–4. Tell students that sometimes local newspapers or magazines ask their readers to vote for their favourite businesses. In this activity, they think about whether this is a good way to decide which businesses are the best or are most popular. (In other words, is this a scientific survey?) Make graph paper, coloured pencils or markers, and scissors available.

Representation: Internalise Comprehension. Activate or supply background knowledge. Provide students with access to blank square grid and bar models to support information processing. Encourage students to annotate diagrams with details to show how each value is represented. For example, number of all people in town, number of newspaper subscribers in town, number of newspaper subscribers who voted.
Supports accessibility for: Visual-spatial processing; Organisation Reading, Speaking: Three Reads. Use this routine to support reading comprehension, without solving, for students. Use the first read to orient students to the situation. Ask students to describe what the situation is about (A town's newspaper ask subscribers to vote on the best restaurant in town). Use the second read to identify quantities and relationships. Ask students what can be counted or measured without focusing on the values (number of people in town, number of newspaper subscribers, number of subscribers who voted, and number of votes for Darnell's). After the third read, ask students to brainstorm possible strategies to solve the problem.
Design Principle(s): Support sense-making; Cultivate conversation; Maximise meta-awareness

Anticipated Misconceptions

Students may wonder how they can answer the question without knowing how many people are in the town. Encourage them to invent a total number of people (such as 100 or 1 000) or to show the percentages as parts of a 10-by-10 square. Remind them that the answer is a percentage, not a number of people. Make sure to discuss the fact that, no matter what the number of people in the town is, the percentage at the end is still the same.

Diagrams may still be too abstract for some. Demonstrate with a large 10-by-10 square: Cut a 5-by-5 square out; this is 25% of the total, representing the subscribers. Put the other 75% aside. Now find 20% of the 5-by-5 square. This is $\frac{1}{5}$ of the square, so is 5 squares. Cut off a strip of 5 to represent the subscribers who voted. Put the rest aside. Now find 80% of 5 squares, which is 4 squares. Cut off and put aside one square. What is left represents 80% of the subscribers who voted. It's only 4% of the people in the town!

Student Task Statement

A town's newspaper held a contest to decide the best restaurant in town. Only people who subscribe to the newspaper can vote. 25% of the people in town subscribe to the newspaper. 20% of the subscribers voted. 80% of the people who voted liked Darnell's BBQ Pit best.

Darnell put a big sign in his restaurant's window that said, "80% say Darnell's is the best!"

Do you think Darnell's sign is making an accurate statement? Support your answer with:

- Some calculations
- An explanation in words
- A diagram that accurately represents the people in town, the newspaper subscribers, the voters, and the people who liked Darnell's best

Student Response

Darnell's sign is very misleading; only 4% of the people in town actually voted for Darnell's.

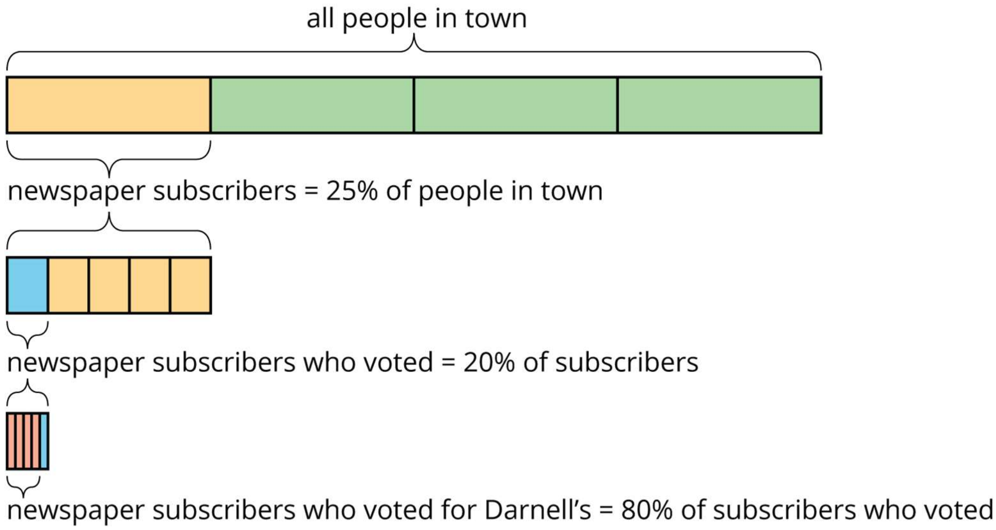
If there are 100 people in the town, then 25 of them subscribe to the paper. 20% of 25 people is 5 subscribers who voted. 80% of 5 is 4 subscribers who voted for Darnell's. So 4% of the people in town thought Darnell's is best. Darnell's sign is misleading. Some correct statements:

- 80% of people who voted for Best Restaurant liked Darnell's best.
- There were 4 times as many people who thought Darnell's was best, compared to all the other restaurants.

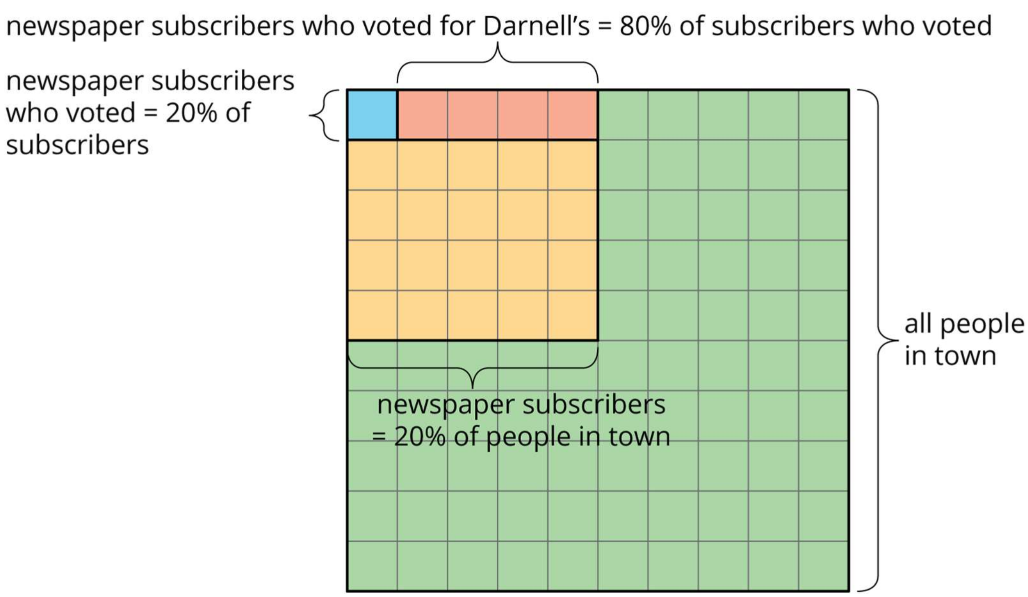
If there are some other number of people in the town, this reasoning still works. One person in the previous calculation now represents a group of $\frac{1}{100}$ of the people in town.

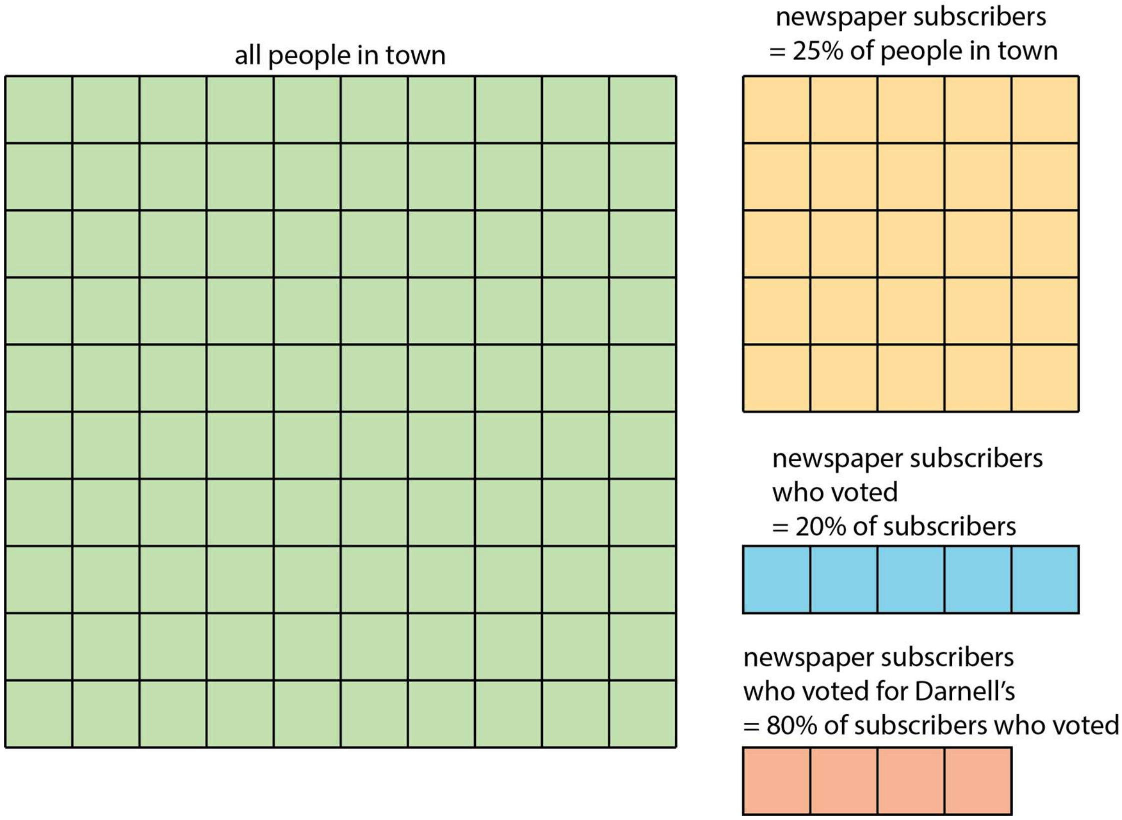
If any students have internalised the fact that to find $n\%$ of something, multiply by $\frac{n}{100}$, then they could reason that 80% of 20% of 25% of the people in town is $(0.80)(0.20)(0.25)$ times the number of people in town. This product is 0.04, or 4%, of the people in town.

A bar model that shows this reasoning should show the fraction or percentages of the whole town and also the percentage of the previous set. An accurate diagram can be drawn on graph paper, or folded from a strip of paper. A sequence of diagrams, such as the pictures shown, are more effective to show the steps of reasoning, as opposed to a single bar model or area diagram.



An area diagram might be on a 10 × 10 grid, each square representing 1% of the town's people.





Activity Synthesis

This problem requires students to revise their idea of what is "the whole" three times: initially, it's the number of people in the town. Then it's the number of subscribers, then the number of voters. "Percentage of what?" is a useful question to ask.

Ask students who chose a specific number of people in the town what percentage they got. Try to find students who chose different numbers of people. All should get 4%.

Choose several diagrams to display and discuss. Ask what part of the diagram represents each quantity:

- all the people in town
- 1% of the people in town
- the people who subscribe to the newspaper (25% of the people in town)
- the people who voted (20% of the subscribers, which is 5% of the people in town)
- the people who voted for Darnell's (80% of those who voted, 4% of the people in town)

Discuss the fact that Darnell's sign is misleading, asking students whom you noticed had interesting or well-stated answers. Students may want to discuss the unfairness of the results when only a few people vote. You might want to consult with a politics/law teacher about actual numbers in a recent election, and whether this is likely to be too controversial to discuss in your class.