The Golden Ratio

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1 Introduction

2 Objectives

Our objectives for this lesson plan are as follows:

1. Have students recognize that we are cultured to find objects that fit the golden ratio appealing.
2. Introduce the properties of golden rectangles and golden triangles and have students prove the existence of golden triangles within a regular pentagon with an inscribed pentagram.

3 Lesson Resources

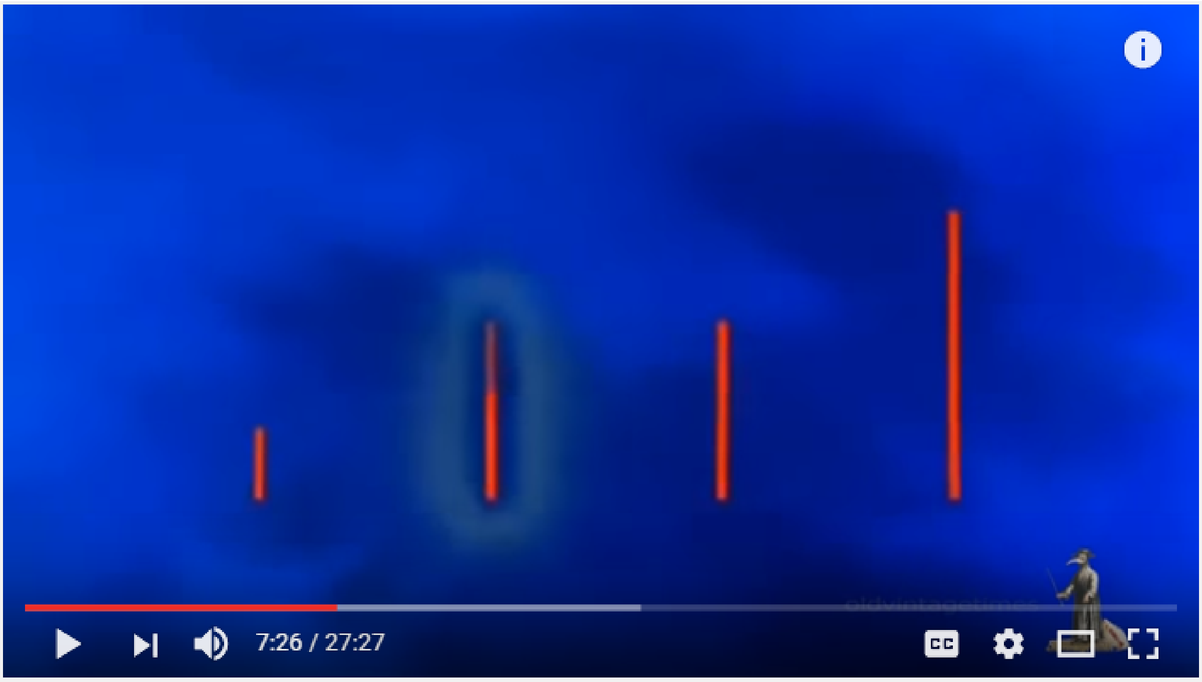
1. Link to Gustav Fechner's study showing that people are cultured to find the golden ratio aesthetically pleasing. `http://goldenratiomyth.weebly.com/phi-in-psychology.html`
2. Donald Duck's Mathemagic Land: `https://www.youtube.com/watch?v=AJgkaU08VvY`
3. (Ab)surd Golden Ratio, TedxTalk Miami by Robb Enzmann: `https://www.youtube.com/watch?v=0vVxL60YFJU&t=307s`

4 Is the Golden Ratio Aesthetically Appealing?

Have students access the National Geographic worksheet in Geogebra. After each student creates a rectangle that is the most aesthetically appealing to them, have the class come together and observe what proportion of students created rectangles with ratios close to the Golden Ratio. Even if the consensus of the classroom do not show an aesthethic preference for Golden Rectangles, you can then show them the website summarizing Gustav Fechner's 19th century study about rectangle preferences. We will begin the next section by introducing Donald Duck's Mathemagic video, which also provides examples of how the Golden Ratio appears in nature and architexture.

5 Golden Ratio

After the National Geographic activity, bring up the Mathemagic video: `https://www.youtube.com/watch?v=AJgkaU08VvY` at 7:00-12:40 minutes.

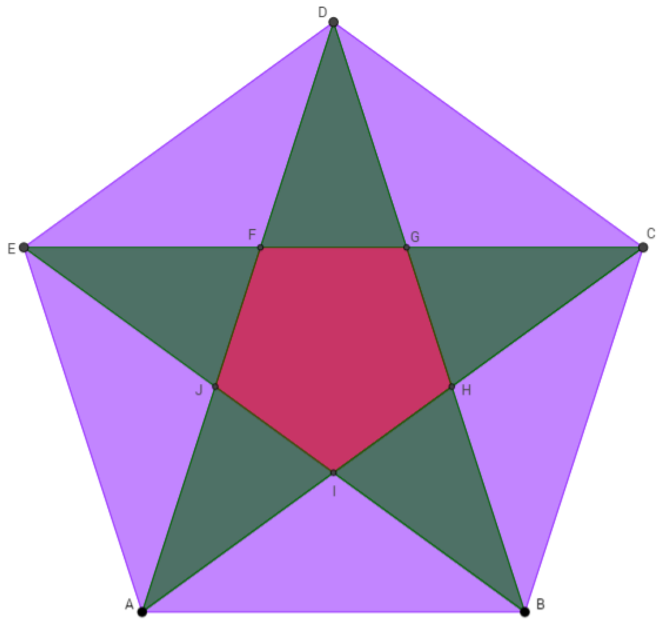


Have students go to the Geogebra worksheet about the Golden Ratio.

6 Regular Pentagon/Golden Triangle

Now that students have learned the definition of the golden ratio, we will progress to the next aspect of the Donald Duck mathemagic video about The Golden Ratio within a Pentagon with an inscribed Pentagram. At the end of this lesson, students should be able to easily construct a Golden Triangle.

Step 2: Have students open up the Geogebra worksheet.



Assume that the length of AB is 1. Have students hypothesize what the ratio $\frac{AC}{AB}$ is. Once they have an estimate, have them prove their answer by calculating the length of AC. Students may use properties of isosceles triangles and law of sines to determine AC.

Now focus on the triangle AFB. Have students estimate the ratio $\frac{AB}{AF}$, and then prove this ratio. One solution: Because Triangle ABF is similar to Triangle ABC, AF and FB will have side lengths $\frac{1}{\phi}$

Lastly, have them focus on triangle IDH. Once they hypothesize the ratio of $\frac{ID}{IH}$, have them prove their conjectures. One solution: $\angle DIH = 72\circ = \angle DHI$. From the previous problem, we know that DI and $DH = \frac{1}{\phi}$. Therefore we can use law of sines to find the length of IH.

$\frac{sin(72)}{\frac{1}{\phi}} = \frac{sin(36)}{IH}$. Solving for IH yields: $IH = \frac{1}{\phi^2}$

Now ask students to construct a golden triangle. What connections can they make between the pentagram and the golden ratio?

7 Using Fibonacci Numbers to Construct a Golden Rectangle

So far, we've learned about an easy way to construct Golden Triangles. Are there any special properties about Golden Rectangles? From the Mathemagic video, we learned about a special property of a golden rectangle, where if you started with a golden rectangle, and divided the rectangle into a square part and a rectangular part, the smaller rectangle would also be a golden rectangle. This process can continue indefinitely, creating a Golden spiral within our golden rectangle.

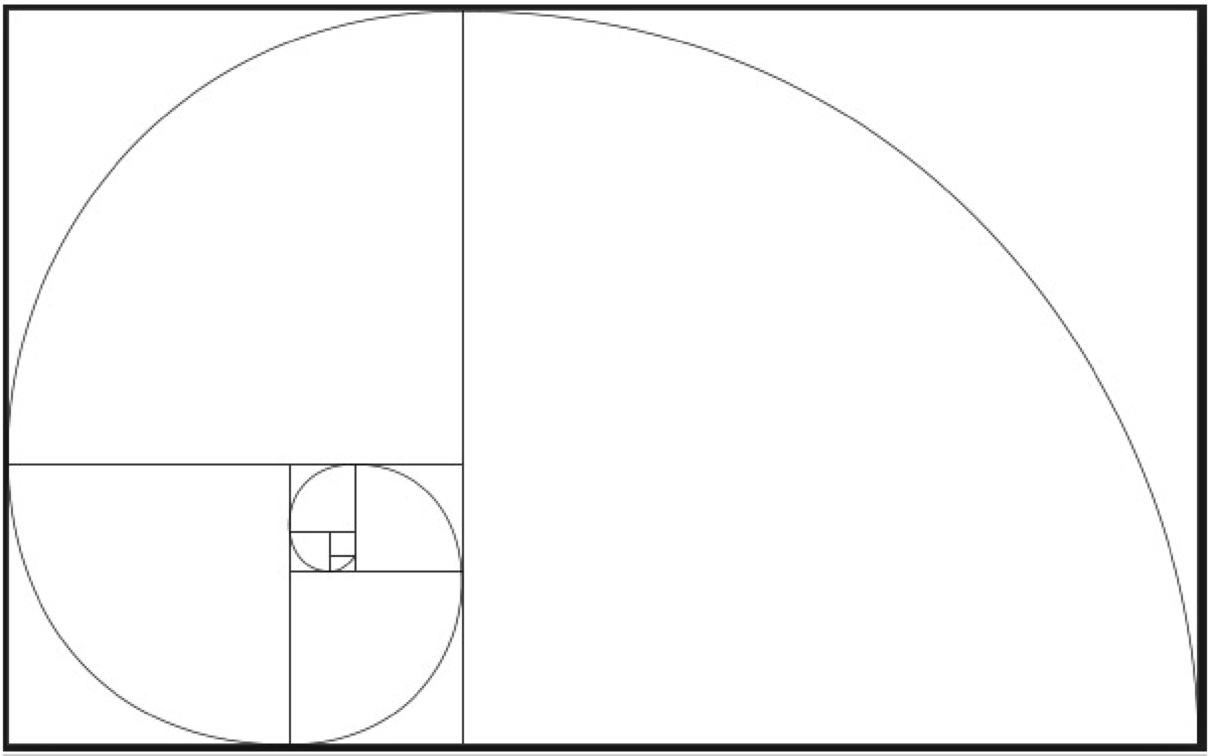
By the end of this lesson, students should be able to answer the following questions:

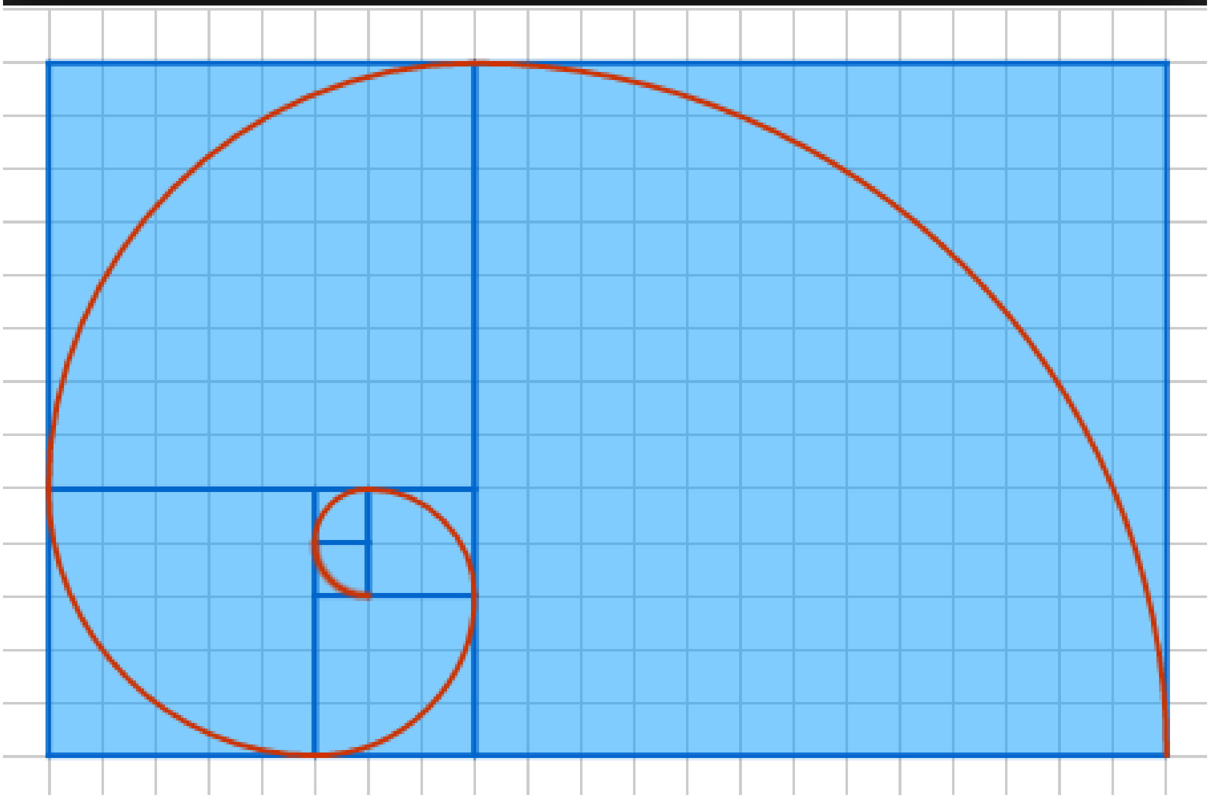
1. Is there any easy way for us to construct a rectangle with such cool properties? How can we do it?
2. Argue why or why not the Fibonacci Spiral is the same as the Golden Spiral.

One way to approximate a Golden Rectangle can be shown from Rob Enzmann's Ted Talk starting from 0:36 seconds until 1:22. URL: `https://www.youtube.com/watch?v=0vVxL60YFJU&t=307s`

After showing the video to the class, introduce the worksheet/animation that helps construct a Golden Rectangle.

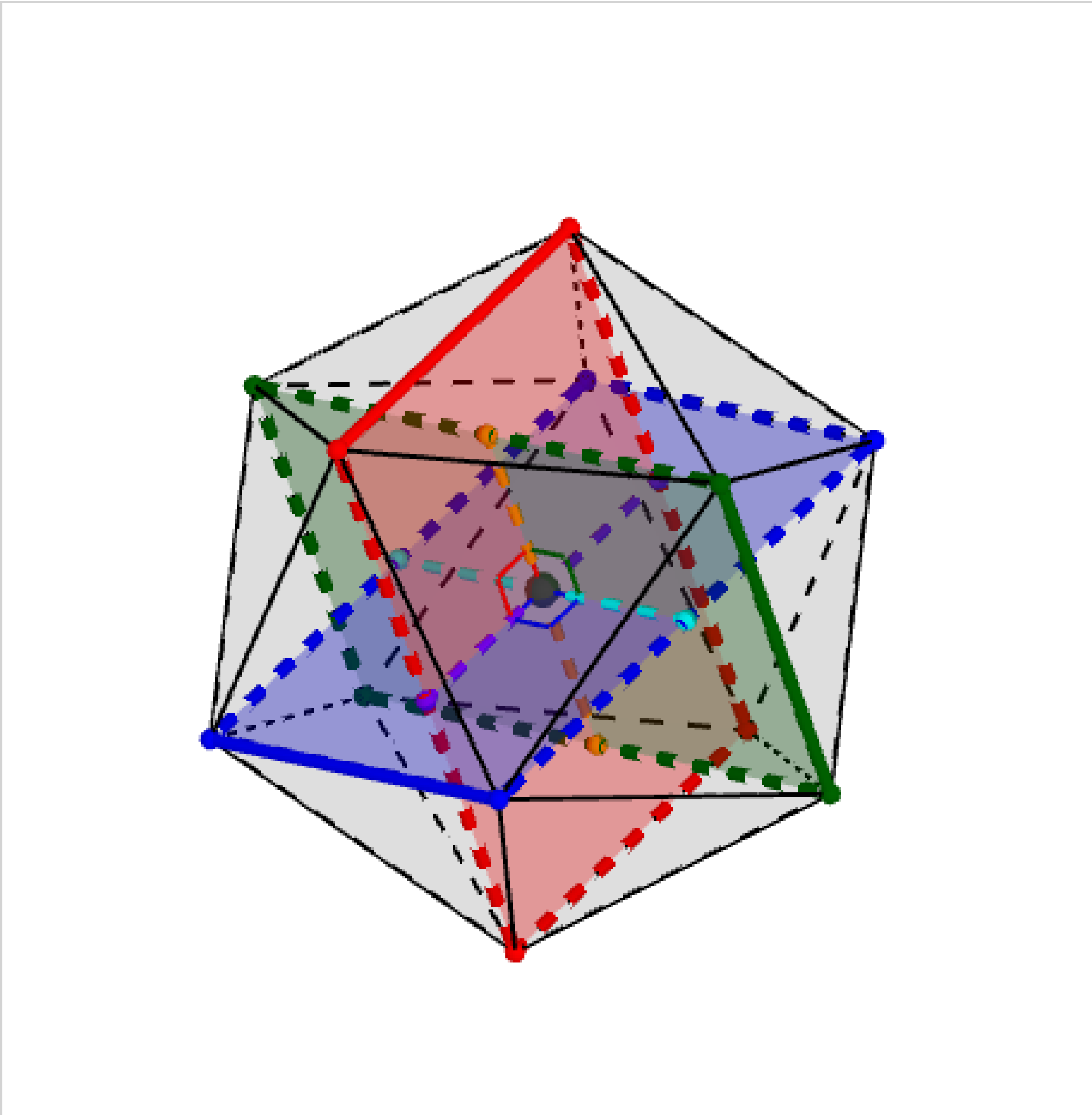
What differences exist between the Golden Rectangle created in Mathemagic Land, and in the rectangle animation from the worksheet?





8 3-D Golden Ratio

We've learned about the special properties of Golden Rectangles and Golden Triangles. But what about 3 dimensions? This last "worksheet" provides an animated example of the Golden Ratio in 3-D space. The objective of this worksheet serves as a glimpse into how the Golden ratio has many facets and is not limited to 2-D examples.



9 Conclusions