Finite Mathematics Objective 1.1

Exercise: Solve the following system of equations. Let z be the parameter.

$$x + 2y + 3z = 16$$
$$2x - y + z = 2$$

(1) $x + 2y + 3z = 16$ ⟶ $x + 2y + 3z = 16$
(2) $2x - y + z = 2$ $2R_2 \to R_2$ $4x - 2y + 2z = 4$

Now we can add (1) and (2):
$$\begin{array}{r} x + 2y + 3z = 16 \\ +\ 4x - 2y + 2z = 4 \\ \hline 5x + 5z = 20 \end{array}$$

We are left with $5x + 5z = 20$. Now we can solve for x (in terms of z) and plug x into (1) or (2).

Solving for x,

$5x + 5z = 20 \Rightarrow 5x = 20 - 5z$ (by subtracting $5z$ from both sides)
$\Rightarrow x = 4 - z$ (by dividing both sides by 5)

So $x = 4 - z$. We can plug into either (1) or (2), but we will just use (1) here:

$x + 2y + 3z = 16$
$4 - z + 2y + 3z = 16$ (as $x = 4 - z$)
$2y + 2z + 4 = 16$ (combining like terms)
$2y + 2z = 16 - 4$ (subtracting $\frac{19}{5}$ from both sides)
NOTE: $16 - 4 = 12$
$2y + 2z = 12$
$2y = 12 - 2z$ (subtracting $2z$ from both sides)
$y = (12 - 2z) \cdot \frac{1}{2}$ (dividing both sides by 2/multiplying both sides by $\frac{1}{2}$)
$y = 12(\frac{1}{2}) - (\frac{1}{2})2z$
$= 6 - z$

Therefore, $x = 4 - z$, $y = 6 - z$